



<https://shikshamentor.com/applied-maths-sem-ii-diploma-msbte-k-scheme-syllabus/>

312301 – Applied Mathematics (Sem II)

**As per MSBTE's K Scheme
CO / CM / IF / AI / AN / DS**

| Unit V | | PROBABILITY DISTRIBUTION | Marks - 12 |
|---------|---|--------------------------|------------|
| Qs. No. | Solution | | |
| 1. | <p>An unbiased coin is tossed 5 times. Find the probability of getting a head.</p> $n = 5, p = \frac{1}{2}, q = \frac{1}{2}, r = 1$ $p(r) = {}^n C_r (p)^r (q)^{n-r}$ $p(1) = {}^5 C_1 \left(\frac{1}{2}\right)^1 \left(\frac{1}{2}\right)^{5-1}$ $= \frac{5}{32} \text{ or } 0.156$ | | |
| 2. | <p>An unbiased coin is tossed 5 times. Find the probability of getting three heads.</p> <div style="border: 1px solid black; padding: 10px; width: fit-content; margin: 10px auto;"> $n = 5, p = \frac{1}{2}, q = \frac{1}{2}, r = 3$ $\therefore P(r) = {}^n C_r p^r q^{n-r}$ $\therefore P(3) = {}^5 C_3 \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^{5-3}$ $\therefore P(3) = \frac{5}{16} \text{ or } 0.3125$ </div> | | |
| 3. | <p>If the coin is tossed three times. Find the probability of getting exactly two Heads.</p> $S = \{HHH, HHT, THH, HTH, HTT, THT, TTH, TTT\}$ $\therefore n(S) = 8$ $A = \{HHT, THH, HTH\}$ $\therefore n(A) = 3$ $\therefore P(A) = \frac{n(A)}{n(S)} = \frac{3}{8} = 0.375$ | | |
| 4. | <p>An unbiased coin is tossed 5 times. Find the probability of getting 2 tails.</p> | | |

| | |
|----|---|
| | <p>Here $n = 5, p = 0.5, q = 0.5, r = 2$</p> $P(x = r) = {}^n C_r p^r q^{n-r}$ $\therefore P(2) = {}^5 C_2 (0.5)^2 (0.5)^3$ $\therefore P(2) = \frac{5}{16} \quad \text{or} \quad 0.3125$ |
| 5. | <p>If two coins tossed simultaneously. Find the probability of getting at least one head.</p> <p>Here, $p = 1/2, q = 1/2$</p> <p>$N = 2, r = \text{at least } 1$</p> <p>By Binomial Distribution,</p> $P(\text{at least } 1) = 1 - p(0)$ $P(\text{at least } 1) = 1 - 0.25$ $P(\text{at least one}) = 0.75$ |
| 6. | <p>Three fair coins are tossed. Find the probability that at least two heads appear.</p> $\therefore n(S) = 8$ <p>at least two heads</p> $A = \{HHH, HTH, HHT, THH\}$ $n(A) = 4$ $\therefore p(A) = \frac{n(A)}{n(S)}$ $\therefore p(A) = \frac{4}{8} = \frac{1}{2} \quad \text{or} \quad 0.5$ |
| 7. | <p>An unbiased coin is tossed 6 times. Find probability of getting 2 Heads.</p> |

| | |
|-----|---|
| | <p>Here $n = 6, p = 0.5, q = 0.5, r = 2$</p> $P(x = r) = {}^n C_r p^r q^{n-r}$ $\therefore P(2) = {}^6 C_2 (0.5)^2 (0.5)^4$ $\therefore P(2) = \frac{15}{64} \quad \text{or} \quad 0.2344$ |
| 8. | <p>If unbiased coin is tossed 6 times, find the probability of getting 4 heads.</p> <p>Here, $p = 1/2, q = 1/2$ $N = 6$ & $r = 4$ By Binomial Distribution, $P(4) = 0.2343$</p> |
| 9. | <p>An unbiased coin is tossed 6 times. Find the probability of getting (i) 2 head (ii) Exactly 4 heads.</p> <p>Here, $p = 1/2, q = 1/2$ $N = 6$</p> <p>i) $r = 2$ By Binomial Distribution, $P(2) = 0.234$</p> <p>ii) $r = 4$ By Binomial Distribution, $P(4) = 0.232$</p> |
| 10. | <p>A person fires 10 shots at target. The probability that any shot will hit the target $3/5$. Find the probability that the target is hit exactly 5 times.</p> $n = 10, p = \frac{3}{5}$ $q = 1 - p = 1 - \frac{3}{5} = \frac{2}{5}$ $r = 5$ $p(r) = {}^n C_r (p)^r (q)^{n-r}$ $p(5) = {}^{10} C_5 \left(\frac{3}{5}\right)^5 \left(\frac{2}{5}\right)^{10-5}$ $= 0.2007$ |

11. If 20% of the bolts produce by a machine are defective. Find the probability that out of 4 bolts drawn,
- One is defective.
 - At the most two are defective.
 - At least one is defective.

Given $p = 20\% = \frac{20}{100} = 0.2, n = 4$ and $q = 1 - p = 0.8$

$$p(r) = {}^n C_r p^r q^{n-r}$$

(1) p (one is defective)

$$= p(1) = 4C_1 (0.2)^1 (0.8)^{4-1}$$

$$= 0.4096$$

(2) p (at the most two are defective.)

$$= p(0) + p(1) + p(2)$$

$$= 4C_0 (0.2)^0 (0.8)^{4-0} + 4C_1 (0.2)^1 (0.8)^{4-1} + 4C_2 (0.2)^2 (0.8)^{4-2}$$

$$= 0.9728$$

12. The probability that a man aged 65 will live to 75 is 0.65.
What is the probability that out of 10 men which are now 65, 7 will live to 75.
Given $p=0.65, q=1-0.65=0.35, n=10, r=7$

$$\therefore p(r) = {}^n C_r (p)^r (q)^{n-r}$$

$$\therefore p(7) = {}^{10} C_7 (0.65)^7 (0.35)^{10-7}$$

$$\therefore p(7) = 0.2522$$

13. The probability that a bomb dropped from a Plane will strike the target is $1/5$. If six bombs are dropped, find the probability that exactly two will strike the target.

Given

$$p = \frac{1}{5} = 0.2, q = 1 - 0.2 = 0.8$$

$$n = 6, r = 2$$

$$\therefore p(r) = {}^n C_r (p)^r (q)^{n-r}$$

$$\therefore p(2) = {}^6 C_2 (0.2)^2 (0.8)^{6-2}$$

$$\therefore p(2) = 0.2458$$

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|-------|---|-------|----|----|----|----|----|----|-------|---|----|----|----|---|---|
| 14. | <p>On an average 2% of the population in an area suffer from T. B. What is the probability that out of 5 persons chosen at random from this area, at least two suffer from T. B.?</p> $n = 5, \quad p = 2\% = \frac{2}{100} = 0.02$ <p>Mean $m = np$</p> $\therefore m = 5 \times 0.02 = 0.1$ $p(r) = \frac{e^{-m} m^r}{r!}$ $\therefore p(\text{atleast two}) = 1 - [p(0) + p(1)]$ $= 1 - \left[\frac{e^{-0.1} (0.1)^0}{0!} + \frac{e^{-0.1} (0.1)^1}{1!} \right]$ $= 0.0047$ | | | | | | | | | | | | | | |
| 15. | <p>10% of the component manufactured by company are defective. If twelve components selected at random, find the probability that at least two will be defective.</p> <p>Given $p = 10\% = \frac{10}{100} = 0.1, n = 12$ and $q = 1 - p = 0.9$</p> $p(r) = {}^n C_r p^r q^{n-r}$ $p(\text{atleast two}) = 1 - [p(0) + p(1)]$ $= 1 - [{}^{12} C_0 (0.1)^0 (0.9)^{12-0} + {}^{12} C_1 (0.1)^1 (0.9)^{12-1}]$ $= 0.3409$ | | | | | | | | | | | | | | |
| 16. | <p>Fit a poisson's distribution for the following observations.</p> <table style="margin-left: 20px;"> <tr> <td>x_i</td> <td>20</td> <td>30</td> <td>40</td> <td>50</td> <td>60</td> <td>70</td> </tr> <tr> <td>f_i</td> <td>8</td> <td>12</td> <td>30</td> <td>10</td> <td>6</td> <td>4</td> </tr> </table> <div style="border: 1px solid black; padding: 10px; margin: 10px 0;"> $\text{Mean} = m = \frac{\sum f_i x_i}{\sum f_i}$ $\therefore m = \frac{20(8) + 30(12) + 40(30) + 50(10) + 60(6) + 70(4)}{8 + 12 + 30 + 10 + 6 + 4}$ $\therefore m = \frac{2860}{70} = 40.85$ <p>Poisson distribution is ,</p> $P(x = r) = \frac{e^{-m} m^r}{r!}$ $\therefore P(r) = \frac{e^{-40.85} (40.85)^r}{r!}$ </div> | x_i | 20 | 30 | 40 | 50 | 60 | 70 | f_i | 8 | 12 | 30 | 10 | 6 | 4 |
| x_i | 20 | 30 | 40 | 50 | 60 | 70 | | | | | | | | | |
| f_i | 8 | 12 | 30 | 10 | 6 | 4 | | | | | | | | | |
| 17. | <p>Assuming that 2 in 10 industrial accidents are due to fatigue. Find the probability that exactly 2 out of 8 accidents will be due to fatigue.</p> <p>Here, $p = 2/10 = 0.2$</p> | | | | | | | | | | | | | | |

$q = 0.5$
 $n = 8, r = 2$
 By Binomial Distribution,
 $P(2) = 0.2936$

18. If 3% of the electric bulbs manufacture by a company are defective. Find the probability that in a sample of 100 bulbs. Exactly 5 bulbs are defective
 (Given $e^{-3} = 0.0497$).

Ans Here, $p = 3\% = 3/100 = 0.03$
 $N = 100$
 $m = np = 100 \times 0.03 = 3$
 By Poisson's Distribution,
 $P(5) = 0.1013$

19. A company manufacture electric motors. The probability that an electric motor is defective is 0.01. What is the probability that a sample of 300 electric Motors will contains exactly 5 defective motors?
 (Given $e^{-3} = 0.0498$)

$p = 0.01, n = 300, r = 5$
 $\therefore m = np = 0.01 \times 300 = 3$

$$p(r) = \frac{e^{-m} \cdot (m)^r}{r!}$$

$$p(5) = \frac{e^{-3} \cdot (3)^5}{5!}$$

$$\begin{aligned}
 p(5) &= \frac{(0.0498) \cdot (3)^5}{5!} \\
 &= 0.1008
 \end{aligned}$$

20. In a sample of 1000 cases the mean of certain test is 14 and standard deviation is 2.5. Assuming the distribution to be normal, find

- How many students score above 18?
- How many students score between 12 and 15? [Given : $A(0.4) = 0.1554$, $A(0.8) = 0.2881$, $A(1.6) = 0.4452$]

Given $\bar{x} = 14$ $\sigma = 2.5$ $N = 1000$

$$(1) z = \frac{x - \bar{x}}{\sigma} = \frac{18 - 14}{2.5} = 1.6$$

$$\begin{aligned}\therefore p(\text{score above } 18) &= A(\text{greater than } 1.6) \\ &= 0.5 - A(1.6) \\ &= 0.5 - 0.4452 = 0.0548\end{aligned}$$

$$\begin{aligned}\therefore \text{No. of students} &= N \cdot p \\ &= 1000 \times 0.0548 = 54.8 \text{ i.e., } 55\end{aligned}$$

$$(2) z = \frac{x - \bar{x}}{\sigma} = \frac{12 - 14}{2.5} = -0.8$$

$$z = \frac{x - \bar{x}}{\sigma} = \frac{15 - 14}{2.5} = 0.4$$

$$\begin{aligned}\therefore p(\text{score between } 12 \text{ and } 15) &= A(-0.8) + A(0.4) \\ &= 0.2881 + 0.1554 \\ &= 0.4435\end{aligned}$$

$$\begin{aligned}\therefore \text{No. of students} &= N \cdot p = 1000 \times 0.4435 \\ &= 443.5 \text{ i.e., } 444\end{aligned}$$

21. If 2% of the electric bulbs manufactured by company are defective, find the probability that in a sample of 100 bulbs,

- i) 3 bulbs are defective,
- ii) At the most two bulbs will be defective. ($e^{-2} = 0.1353$)

$$p = 2\% = 0.02, \quad n = 100$$

$$\therefore \text{mean } m = np$$

$$\therefore m = 100 \times 0.02 = 2$$

Poisson's distribution is,

$$P(r) = \frac{e^{-m} \cdot m^r}{r!}$$

(i) 3 bulbs are defective $\therefore r = 3$

$$\therefore P(3) = \frac{e^{-2} (2)^3}{3!}$$

$$\therefore P(3) = 0.1804$$

(ii) At the most two bulbs will be defective $\therefore r = 0, 1, 2$

$$\therefore P(r) = P(0) + P(1) + P(2)$$

$$\begin{aligned}\therefore P(0) &= \frac{e^{-2} (2)^0}{0!} + \frac{e^{-2} (2)^1}{1!} + \frac{e^{-2} (2)^2}{2!} \\ &= 0.6767\end{aligned}$$

22.

In a test on 2000 electric bulbs, it was found that the life of particular make was normally distributed with average life of 2040 hours and standard deviation of 60 hours. Estimate the number of bulbs likely to burn for:

- i) Between 1920 hours and 2160 hours.
 ii) More than 2150 hours. Given that: $A(2) = 0.4772$
 $A(1.83) = 0.4664$

Given $\bar{x} = 2040$ $\sigma = 60$ $N = 2000$

i) For $x = 1920$

$$z = \frac{x - \bar{x}}{\sigma} = \frac{1920 - 2040}{60} = -2$$

For $x = 2160$

$$z = \frac{x - \bar{x}}{\sigma} = \frac{2160 - 2040}{60} = 2$$

$$\begin{aligned} \therefore p(\text{between } 1920 \text{ and } 2160) &= A(\text{between } -2 \text{ and } 2) \\ &= A(-2) + A(2) \\ &= 0.4772 + 0.4772 \\ &= 0.9544 \end{aligned}$$

$$\begin{aligned} \therefore \text{No. of bulbs} &= N \cdot p \\ &= 2000 \times 0.9544 = 1908.8 \approx 1909 \end{aligned}$$

ii) For $x = 2150$

$$z = \frac{x - \bar{x}}{\sigma} = \frac{2150 - 2040}{60} = 1.83$$

$$\begin{aligned} \therefore p(\text{more than } 2150) &= A(\text{more than } 1.83) \\ &= 0.5 - A(1.83) \\ &= 0.5 - 0.4664 \end{aligned}$$

$$\therefore p(\text{more than } 2150) = 0.0336$$

$$\begin{aligned} \therefore \text{No. of students} &= N \cdot p = 2000 \times 0.0336 \\ &= 67.2 \approx 67 \end{aligned}$$

23.

If the probability of a bad reaction from the certain injection is 0.001, determine the chance that out of 2000 individuals more than two will get a bad reaction. (Given $e^2 = 7.4$)

$$p = 0.001, n = 2000$$

$$\therefore m = np = 0.001 \times 2000 = 2$$

$$p(\text{more than } 2) = p(3) + p(4) + p(5) + \dots$$

$$= 1 - [p(0) + p(1) + p(2)]$$

$$= 1 - \left[\frac{e^{-2} \cdot (2)^0}{0!} + \frac{e^{-2} \cdot (2)^1}{1!} + \frac{e^{-2} \cdot (2)^2}{2!} \right]$$

$$= 0.3233$$

24.

In a sample of 1000 cases the mean of certain test is 14 and S.D is 2.5. Assuming the distribution to be normal. Find

i) How many students score between 12 and 15?

ii) How many students score above 18?

[Given : $A(0.8) = 0.2881$, $A(0.4) = 0.1554$, $A(1.6) = 0.4452$]

$$\text{Given } \bar{x} = 14 \quad \sigma = 2.5 \quad N = 1000$$

$$(1) z = \frac{x - \bar{x}}{\sigma} = \frac{18 - 14}{2.5} = 1.6$$

$$\begin{aligned} \therefore p(\text{score above } 18) &= A(\text{greater than } 1.6) \\ &= 0.5 - A(1.6) \\ &= 0.5 - 0.4452 = 0.0548 \end{aligned}$$

$$\begin{aligned} \therefore \text{No. of students} &= N \cdot p \\ &= 1000 \times 0.0548 = 54.8 \text{ i.e., } 55 \end{aligned}$$

$$(2) z = \frac{x - \bar{x}}{\sigma} = \frac{12 - 14}{2.5} = -0.8$$

$$z = \frac{x - \bar{x}}{\sigma} = \frac{15 - 14}{2.5} = 0.4$$

$$\begin{aligned} \therefore p(\text{score between } 12 \text{ and } 15) &= A(-0.8) + A(0.4) \\ &= 0.2881 + 0.1554 \\ &= 0.4435 \end{aligned}$$

$$\begin{aligned} \therefore \text{No. of students} &= N \cdot p = 1000 \times 0.4435 \\ &= 443.5 \text{ i.e., } 444 \end{aligned}$$

25.

The number of road accidents met with by taxi drivers follow Poisson distribution with mean 2 out of 5000 taxi in the city, find the number of drivers.

i) Who does not meet an accident?

Who met with an accidents more than 3 items? (Given $e^{-2} = 0.1353$)

$$\text{Let } N = 5000, \text{ Mean } m = 2$$

$$p(r) = \frac{e^{-m} m^r}{r!}$$

$$(i) r = 0 \quad \therefore p(0) = \frac{e^{-2} 2^0}{0!}$$

$$\therefore p(0) = 0.1353$$

$$\text{Number of taxi drivers} = N \times p = 5000 \times 0.1353 = 676.5 \cong 677$$

(ii) More than three

$$= 1 - \left[\frac{e^{-2} 2^0}{0!} + \frac{e^{-2} 2^1}{1!} + \frac{e^{-2} 2^2}{2!} + \frac{e^{-2} 2^3}{3!} \right]$$

$$= 0.1429$$

$$\text{Number of taxi drivers} = N \times p = 5000 \times 0.1429 = 714.5 \approx 715$$

24.

In a sample of 1000 cases the mean of certain test is 14 and S.D is 2.5. Assuming the distribution to be normal. Find

i) How many students score between 12 and 15?

ii) How many students score above 18?

[Given : $A(0.8) = 0.2881$, $A(0.4) = 0.1554$, $A(1.6) = 0.4452$]

$$\text{Given } \bar{x} = 14 \quad \sigma = 2.5 \quad N = 1000$$

$$(1) z = \frac{x - \bar{x}}{\sigma} = \frac{18 - 14}{2.5} = 1.6$$

$$\begin{aligned} \therefore p(\text{score above } 18) &= A(\text{greater than } 1.6) \\ &= 0.5 - A(1.6) \\ &= 0.5 - 0.4452 = 0.0548 \end{aligned}$$

$$\begin{aligned} \therefore \text{No. of students} &= N \cdot p \\ &= 1000 \times 0.0548 = 54.8 \text{ i.e., } 55 \end{aligned}$$

$$(2) z = \frac{x - \bar{x}}{\sigma} = \frac{12 - 14}{2.5} = -0.8$$

$$z = \frac{x - \bar{x}}{\sigma} = \frac{15 - 14}{2.5} = 0.4$$

$$\begin{aligned} \therefore p(\text{score between } 12 \text{ and } 15) &= A(-0.8) + A(0.4) \\ &= 0.2881 + 0.1554 \\ &= 0.4435 \end{aligned}$$

$$\begin{aligned} \therefore \text{No. of students} &= N \cdot p = 1000 \times 0.4435 \\ &= 443.5 \text{ i.e., } 444 \end{aligned}$$

25.

The number of road accidents met with by taxi drivers follow Poisson distribution with mean 2 out of 5000 taxi in the city, find the number of drivers.

i) Who does not meet an accident?

Who met with an accidents more than 3 items? (Given $e^{-2} = 0.1353$)

$$\text{Let } N = 5000, \text{ Mean } m = 2$$

$$p(r) = \frac{e^{-m} m^r}{r!}$$

$$(i) r = 0 \quad \therefore p(0) = \frac{e^{-2} 2^0}{0!}$$

$$\therefore p(0) = 0.1353$$

$$\text{Number of taxi drivers} = N \times p = 5000 \times 0.1353 = 676.5 \cong 677$$

(ii) More than three

$$= 1 - \left[\frac{e^{-2} 2^0}{0!} + \frac{e^{-2} 2^1}{1!} + \frac{e^{-2} 2^2}{2!} + \frac{e^{-2} 2^3}{3!} \right]$$

$$= 0.1429$$

$$\text{Number of taxi drivers} = N \times p = 5000 \times 0.1429 = 714.5 \approx 715$$

$$\text{Number of taxi drivers} = N \times p = 5000 \times 0.1429 = 714.5 \approx 715$$

26. Weight of 4000 students are found to be normally distributed with mean 50 kgs and standard deviation 5 kgs.

Find the number of students with weights

- i) less than 45 kgs
- ii) between 45 and 60 kgs

(Given : For a standard normal variate z area under the curve between $z = 0$ and $z = 1$ is 0.3413 and that between $z = 0$ and $z = 2$ is 0.4772)

$$\text{Given } \bar{x} = 50, \sigma = 5, N = 4000$$

$$(i) \text{ For } x = 45, z = \frac{x - \bar{x}}{\sigma} = \frac{45 - 50}{5} = -1$$

$$\begin{aligned} \therefore p(\text{less than } 45) &= A(\text{less than } -1) \\ &= 0.5 - A(1) \\ &= 0.5 - 0.3413 \\ &= 0.1587 \end{aligned}$$

$$\begin{aligned} \therefore \text{No. of students} &= N \cdot p \\ &= 4000 \times 0.1587 = 634.8 \text{ i.e., } 635 \end{aligned}$$

$$(ii) \text{ For } x = 45, z = \frac{x - \bar{x}}{\sigma} = \frac{45 - 50}{5} = -1$$

$$\text{For } x = 60, z = \frac{x - \bar{x}}{\sigma} = \frac{60 - 50}{5} = 2$$

$$\begin{aligned} \therefore p(\text{ between } 45 \text{ and } 60) &= A(-1) + A(2) \\ &= 0.3413 + 0.4772 \\ &= 0.8185 \end{aligned}$$

$$\begin{aligned} \therefore \text{No. of students} &= N \cdot p = 4000 \times 0.8185 \\ &= 3274 \end{aligned}$$

27. If 2% of the electric bulbs manufactured by company are defective, find the probability that in a sample of 100 bulbs,

- i) 3 bulbs are defective,
- ii) At least two are defective.

$$p = 2\% = 0.02, n = 100$$

$$\therefore \text{mean } m = np$$

$$\therefore m = 100 \times 0.02 = 2$$

Poisson's distribution is,

$$P(r) = \frac{e^{-m} \cdot m^r}{r!}$$

$$(i) 3 \text{ bulbs are defective } \therefore r = 3$$

$$\therefore P(3) = \frac{e^{-2} (2)^3}{3!}$$

$$\therefore P(3) = 0.1804$$

(ii) At least two are defective

$$\therefore P(\text{at least two are defective}) = 1 - [P(0) + P(1)]$$

$$\begin{aligned} \therefore P(\text{at least two are defective}) &= 1 - \left[\frac{e^{-2} (2)^0}{0!} + \frac{e^{-2} (2)^1}{1!} \right] \\ &= 0.5939 \end{aligned}$$

28.

In a sample of 1000 cases the mean of certain test is 14 and standard deviation is 2.5. Assuming the distribution to be normal, find

iii) How many students score above 18?

iv) How many students score between 12 and 15? Given

$$\text{Frequency 0 to 0.8} = 0.2881$$

$$\text{Frequency 0 to 0.4} = 0.1554$$

$$\text{Frequency 0 to 1.6} = 0.4452.$$

$$\text{Given } \bar{x} = 14 \quad \sigma = 2.5 \quad N = 1000$$

$$(i) z = \frac{x - \bar{x}}{\sigma} = \frac{18 - 14}{2.5} = 1.6$$

$$\therefore p(\text{score above 18}) = A(\text{greater than 1.6})$$

$$= 0.5 - A(1.6)$$

$$= 0.5 - 0.4452 = 0.0548$$

$$\therefore \text{No. of students} = N \cdot p$$

$$= 1000 \times 0.0548 = 54.8 \text{ i.e., } 55$$

$$(2) z = \frac{x - \bar{x}}{\sigma} = \frac{12 - 14}{2.5} = -0.8$$

$$z = \frac{x - \bar{x}}{\sigma} = \frac{15 - 14}{2.5} = 0.4$$

$$\begin{aligned} \therefore p(\text{score between 12 and 15}) &= A(-0.8) + A(0.4) \\ &= 0.2881 + 0.1554 \\ &= 0.4435 \end{aligned}$$

$$\begin{aligned} \therefore \text{No. of students} &= N \cdot p = 1000 \times 0.4435 \\ &= 443.5 \text{ i.e., } 444 \end{aligned}$$

29. In a certain examination 500 students appeared. Mean score is 68 with S.D. 8. Find the number of students scoring

- i) Less than 50
- ii) Scoring more than 60

Given, Area between $z = 0$ and $z = 2.25$ is 0.4878
Area between $z = 0$ and $z = 1$ is 0.3413

Given $\bar{x} = 68$ $\sigma = 8$ $N = 500$

$$i) z = \frac{x - \bar{x}}{\sigma} = \frac{50 - 68}{8} = -2.25$$

$$\begin{aligned} \therefore p(\text{Less than 50}) &= A(\text{less than } -2.25) \\ &= 0.5 - A(z = 0 \text{ to } z = 2.25) \\ &= 0.5 - 0.4878 \\ &= 0.0122 \end{aligned}$$

$$\begin{aligned} \therefore \text{No. of students} &= N \cdot p \\ &= 500 \times 0.0122 = 6.1 \text{ i.e., } 6 \end{aligned}$$

$$ii) z = \frac{x - \bar{x}}{\sigma} = \frac{60 - 68}{8} = -1$$

$$\begin{aligned} \therefore p(\text{More than 60}) &= A(\text{more than } -1) \\ &= A(z = 0 \text{ to } z = 1) + 0.5 \\ &= 0.3413 + 0.5 \\ &= 0.8413 \end{aligned}$$

$$\begin{aligned} \therefore \text{No. of students} &= N \cdot p = 500 \times 0.8413 \\ &= 420.65 \text{ i.e., } 421 \end{aligned}$$

30. If 30% of the bulbs produced are defective, find the probability that out of 4 bulbs selected

- i) One is defective
- ii) at the most two are defective

$$p = 30\% = \frac{30}{100} = 0.3 \quad q = 1 - 0.3 = 0.7$$

$$n = 4$$

$$(i) \because p(r) = {}^n C_r (p)^r (q)^{n-r}$$

$$\therefore p(1) = {}^4 C_1 (0.3)^1 (0.7)^{4-1} = 0.4116$$

ii)

r = at most 2 are defective.

$$P(\text{at most 2}) = p(0) + p(1) + p(2)$$

$$P(\text{at most 2}) = 0.2401 + 0.4116 + 0.2646$$

$$P(\text{at most 2}) = 0.9163$$

| | |
|-----|---|
| 31. | <p>I.Q.'s are normally distributed with mean 100 and standard deviation 15. Find the probability that a randomly selected person has</p> <p>i) An I.Q more than 130 ii) An I.Q. between 85 and 115.</p> <p>Given [z = 2, Area = 0.4772 and z = 1, Area = 0.3413]</p> <p>Given $\bar{x} = 100$ $\sigma = 15$</p> <p>i) $z = \frac{130-100}{15} = 2$</p> <p>$\therefore P(130 \leq x) = P(2 \leq z)$</p> <p>$= 0.5 - P(0 \leq z \leq 2)$</p> <p>$= 0.5 - 0.4772$</p> <p>$= 0.0228$</p> <p>2) $z = \frac{85-100}{15} = -1$ $z = \frac{115-100}{15} = 1$</p> <p>$\therefore P(85 \leq x \leq 115) = P(-1 \leq z \leq 1)$</p> <p>$= P(-1 \leq z \leq 0) + P(0 \leq z \leq 1)$</p> <p>$= 0.3413 + 0.3413$</p> <p>$= 0.6826$</p> |
| 32. | <p>3% of a given lot of manufactured parts are defective. What is the probability that in a sample of 4 items none will be defective?</p> <p>Here $p = 3\% = 0.03$ $q = 0.97$ $n = 4$ & $r = 0$ By Binomial Distribution, $P(0) = 0.8853$</p> |
| 33. | <p>In a town 10 accidents took place in a period of 50 days. Assuming Poisson distribution, find the probability that there will be 3 or more accidents per day.</p> <p>$P(3 \text{ or more}) = 0.012$</p> |
| 34. | <p>The probability of getting an item defective is 0.005. What is the probability that exactly 3 items in a sample of 200 are defective? (Given $e^{-1} = 0.3679$)</p> <p>Here, $p = 0.005$ $N = 200$ $M = 1$ By Poisson's Distribution, $P(3) = 0.06131$</p> |

$$P(3) = 0.06131$$

35. Fit a Poisson distribution to set of observations

| | | | | | |
|-------|-----|----|----|---|---|
| X_i | 0 | 1 | 2 | 3 | 4 |
| F_i | 122 | 60 | 15 | 2 | 1 |

$$P(r) = \frac{e^{-m} m^r}{r!} = \frac{e^{-0.684} (0.684)^r}{r!}$$

36. If 5% of the electric bulbs manufacturing by a company are defective, use Poisson distribution to find the probability that in a sample of 100 bulbs.

- i) None is defective
- ii) Five bulbs are defective ($e^{-5} = 0.007$)

Given $p = 5\% = 0.05$

$$n = 100$$

$$\text{mean } m = np = 100 \times 0.05 \quad \therefore m = 5$$

$$P(r) = \frac{e^{-m} m^r}{r!}$$

i) None is defective

$$r = 0$$

$$P(0) = \frac{e^{-5} (5)^0}{0!}$$

$$P(0) = 0.007$$

ii) Five bulbs are defective

$$r = 5$$

$$P(5) = \frac{e^{-5} (5)^5}{5!}$$

$$P(5) = 0.1823$$

37. A problem is given to the three students Ram, Shyam and Amit, whose chances of solving it are $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{4}$ respectively. If they attempt to solve a problem independently, Find the probability that the problem is solved by at least one of them.

(Not in Syllabus)

38. In 200 sets of tosses of 5 fair coins, in how many ways you can expect.
- at least two heads
 - at the most two heads

$$p = \frac{1}{2}, q = \frac{1}{2}$$

$$n = 5$$

$$p(r) = {}^n C_r p^r q^{n-r}$$

i) at least two heads

$$P(r) = 1 - [p(0) + p(1)]$$

$$= 1 - \left[{}^5 C_0 \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^{5-0} + {}^5 C_1 \left(\frac{1}{2}\right)^1 \left(\frac{1}{2}\right)^{5-1} \right] = 1 - 0.1875$$

$$= \frac{13}{16} \text{ or } 0.8125$$

$$\text{No. of ways} = 200 \times 0.8125 = 162.5 \approx 163$$

ii) At the most two heads

$$P(r) = p(0) + p(1) + p(2)$$

$$P(r) = p(0) + p(1) + p(2)$$

$$= 0.1875 + {}^5 C_2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^{5-2}$$

$$= 0.5$$

$$\text{No. of ways} = 200 \times 0.5 = 100$$

39. The probability that a machine manufactured by a company will be defective $\frac{1}{10}$. If 5 such machines are manufactured find probability that.
- Exactly two will be defective.
 - At least two will be defective.

Binomial distribution is , $P(x=r) = {}^nC_r (p)^r (q)^{n-r}$

(i) Exactly two will be defective

$$\begin{aligned}\therefore p(2) &= {}^5C_2 (0.1)^2 (0.9)^{5-2} \\ &= 0.0729\end{aligned}$$

(ii) At Least two will be defective

i.e $r = 2$ or 3 or 4 or 5

$$\begin{aligned}\therefore p(r=2 \text{ or } 3 \text{ or } 4 \text{ or } 5) &= 1 - p(r=0 \text{ or } 1) \\ &= 1 - (p(0) + p(1)) \\ &= 1 - [{}^5C_0 (0.1)^0 (0.9)^{5-0} + {}^5C_1 (0.1)^1 (0.9)^4] \\ &= 0.08146\end{aligned}$$

Fit a Poisson's distribution for the following observations

| | | | | | | |
|-----|-------|----|----|---|---|---|
| 40. | x_i | 0 | 1 | 2 | 3 | 4 |
| | f_i | 21 | 18 | 7 | 3 | 1 |

$$\text{Mean} = m = \frac{\sum f_i x_i}{\sum f_i}$$

$$\therefore m = \frac{0(21) + 1(18) + 2(7) + 3(3) + 4(1)}{21 + 18 + 7 + 3 + 1}$$

$$\therefore m = \frac{45}{50} = \frac{9}{10} = 0.9$$

Poisson distribution is ,

$$P(x=r) = \frac{e^{-m} m^r}{r!}$$

$$\therefore P(r) = \frac{e^{-0.9} (0.9)^r}{r!}$$

41. A room has 3 electrical lamps. From a collection of 15 electric bulbs of which only 10 are good, 3 are selected at random and put in the lamps. Find the probability that the room is lighted by at least one of the bulbs.

$$P = P(\text{room is lighted by atleast one of the bulbs})$$

$$P = 1 - P(\text{room is not lighted})$$

$$P = 1 - \frac{{}^5C_3}{{}^{15}C_3}$$

$$P = \frac{89}{91} \text{ or } 0.9780$$

OR

$$P = P(\text{room is lighted by atleast one of the bulbs})$$

$$P = \frac{{}^{10}C_1 \times {}^5C_2 + {}^{10}C_2 \times {}^5C_1 + {}^{10}C_3}{{}^{15}C_3}$$

$$P = \frac{89}{91} \text{ or } 0.9780$$