SHIKSHA MENTOR THE PERFECT MENTOR FOR YOUR BRIGHT CAREER	https://shikshamentor.com/applied-maths-sem-ii-diploma-msbte-k-scheme-syllabus/ 312301 – Applied Mathematics (Sem II) As per MSBTE's K Scheme CO / CM / IF / AI / AN / DS

Unit IV	NUMERICAL METHODS	Marks - 14

Qu. No.	Solution
1	Show that there exist a root of the equation $x^2-2x-1=0$ in $(-1,0)$ and find approximate value of the root by using Bisection method. (Use two iterations) $x^2-2x-1=0$ $f(x)=x^2-2x-1$ $f(-1)=2$ $f(0)=-1$ root is in $(-1,0)$ $\therefore x_1=\frac{-1+0}{2}=-0.5$ $\therefore f(-0.5)=0.25$ $\therefore$ root is in $(-0.5,0)$ $\therefore x_2=\frac{-0.5+0}{2}=-0.25$
2	Solve the following system of by equtions by Jacobi's -Iteration method. (Two iterations) $5x+2y+z=12$, $x+4y+2z=15$, $x+2y+5z=20$ $x=\frac{1}{5}(12-2y-z)$ $y=\frac{1}{4}(15-x-2z)$ $z=\frac{1}{5}(20-x-2y)$ Starting with $x_0=y_0=z_0=0$ $x_1=2.4$ $y_1=3.75$ $z_1=4$ $x_2=0.1$ $y_2=1.15$ $z_2=2.02$
3	

	Solve the following system of equation by using Gauss-Seidel method. (Two iterations) $15x+2y+z=18$, $2x+20y-3z=19$, $3x-6y+25z=22$ $x=\frac{1}{15}(18-2y-z)$ $y=\frac{1}{20}(19-2x+3z)$ $z=\frac{1}{25}(22-3x+6y)$ Starting with $y_0=z_0=0$ $x_1=1.2$ $y_1=0.83$ $z_1=0.935$ $x_2=1.027$ $y_2=0.988$ $z_2=0.994$
4	Find the approximate root of the equation $x^4-x-10=0$, by Newton-Raphson method (Carry out four iterations) Let $f(x)=x^4-x-10$ $f(1)=-10<0$ $f(2)=4>0$ $f'(x)=4x^3-1$ Initial root $x_0=2$ $\therefore f'(2)=31$ $x_1=x_0-\frac{f(x_0)}{f'(x_0)}=2-\frac{f(2)}{f'(2)}=1.871$ $x_2=1.871-\frac{f(1.871)}{f'(1.871)}=1.856$ $x_3=1.856-\frac{f(1.856)}{f'(1.856)}=1.856$ $x_4=1.856-\frac{f(1.856)}{f'(1.856)}=1.856$ OR
5	Let $f(x)=x^2+x-3$ $f(1)=-1$ $f(2)=3$ $\therefore$ the root is in $(1,2)$ $x_1=\frac{a+b}{2}=\frac{1+2}{2}=1.5$ $f(1.5)=0.75>0$ $x_2=\frac{x_1+a}{2}=\frac{1.5+1}{2}=1.25$

6	$5x - 2y + 3z = 18;$ $x + 7y - 3z = 22$, $2x - y + 6z = 22$ $5x - 2y + 3z = 18;$ $x + 7y - 3z = 22$, $2x - y + 6z = 22$ $x = \frac{1}{5}(18 + 2y - 3z)$ $y = \frac{1}{7}(22 - x + 3z)$ $z = \frac{1}{6}(22 - 2x + y)$ Starting with $x_0 = y_0 = z_0 = 0$ $x_1 = 3.6$ $y_1 = 2.629$ $z_1 = 2.905$ $x_2 = 2.909$ $y_2 = 3.972$ $z_2 = 3.359$
7	Using Newton-Raphson method to find the approximate value of $\sqrt[3]{100}$ (perform 4 iterations) Let $x = \sqrt[3]{100}$ $\therefore x^3 - 100 = 0$ $f(x) = x^3 - 100$ $f(4) = -36 < 0$ $f(5) = 25 > 0$ $f'(x) = 3x^2$ Initial root x_0=5 $\therefore f'(5) = 75$ $x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 5 - \frac{f(5)}{f'(5)} = 4.6667$ $x_2 = 4.6667 - \frac{f(4.6667)}{f'(4.6667)} = 4.6417$ $x_3 = 4.6417 - \frac{f(4.6417)}{f'(4.6417)} = 4.6416$ $x_4 = 4.6416 - \frac{f(4.6416)}{f'(4.6416)} = 4.6416$
8	

	Using Bisection method find approximate root of the equation $x^3-4x-9=0$ $Let\ f(x)=x^3-4x-9$ $f(2)=-9<0$ $f(3)=6>0$ $\therefore$ the root is in $(2,3)$ $x_1=\frac{a+b}{2}=\frac{2+3}{2}=2.5$ $f(2.5)=-3.375<0$ $\therefore$ the root is in (2.5,3) $x_2=\frac{x_1+b}{2}=\frac{2.5+3}{2}=2.75$ $f(2.75)=0.797>0$ the root is in $(2.5,2.75)$ $x_3=\frac{x_1+x_2}{2}=\frac{2.5+2.75}{2}=2.625$
9	Find the Root of equation F(x)=cosx-xe^x=0 by using Regula Falsi method (carry out two iteration). Ist iteration- X_1=0.31467 F(X_1)=0.51987 IInd iteration- X_2=0.44673

10	Solve the following equations by Jacobi's method, performing three iterations only: $20x+y-2z=17,\quad 3x+20y-z=-18,\quad 2x-3y+20z=25$ $20x+y-2z=17$ $3x+20y-z=-18$ $2x-3y+20z=25$ $\therefore x=\frac{1}{20}(17-y+2z)$ $y=\frac{1}{20}(-18-3x+z)$ $z=\frac{1}{20}(25-2x+3y)$ Starting with $x_0=0=y_0=z_0$ $x_1=0.85$ $y_1=-0.9$ $z_1=1.25$ $x_2=1.02$ $y_2=-0.965$ $z_2=1.03$ $x_3=1.001$ $y_3=-1.002$ $z_3=1.003$
11	$f(x)=x\log_{10}x-1.2$ $f(0)=-1.2<0$ $f(1)=-1.2<0$ $f(2)=-0.6<0$ $f(3)=0.231>0$ $\therefore Initial\ Root, x_0=3$ $Now, f(x)=x\frac{logx}{log10}-1.2$ $f'(x)=\frac{d[\frac{xlogx}{log10}]}{dx}-\frac{d(1.2)}{dx}$

	$f'(x) = \frac{1}{log10}\frac{d(xlogx)}{dx} - 0$ $f'(x) = \frac{1}{log10}[x\frac{1}{x} + logx.1]$ $f'(x) = \frac{1}{log10}[1 + logx]$ Ist Iteration: $x_1 = 2.83$ IInd Iteration: $x_2 = 2.74$ IIIrd Iteration: $x_3 = 1.36$
12	Show that the root of $x^3 - 9x + 1 = 0$ lies between 2 and 3 $f(x) = x^3 - 9x + 1$ $f(2) = (2)^3 - 9(2) + 1 = -9 < 0$ $f(3) = (3)^3 - 9(3) + 1 = 1 > 0$ $\therefore$ Root lies between 2 and 3
13	$5x - 2y + 3z = 18,\ x + 7y - 3z = -22,\ 2x - y + 6z = 22$ $x = \frac{1}{5}(18 + 2y - 3z)$ $y = \frac{1}{7}(-22 - x + 3z)$ $z = \frac{1}{6}(22 - 2x + y)$ Starting with $x_0 = y_0 = z_0 = 0$ $x_1 = 3.6$ $y_1 = -3.657$ $z_1 = 1.857$ $x_2 = 1.023$ $y_2 = -2.493$ $z_2 = 2.910$ $x_3 = 0.857$ $y_3 = -2.018$ $z_3 = 3.045$ $x_4 = 0.966$ $y_4 = -1.976$ $z_4 = 3.015$

14	Using Newton-Raphson method find the approximate root of the equation correct upto 3 places of decimals. $x^3 - 2x - 5 = 0$ (Use four iterations) $f(x) = x^3 - 2x - 5$ $f(2) = -1 < 0$ $f(3) = 16 > 0$ Root is in $(2,3)$ $f'(x) = 3x^2 - 2$ Initial root $x_0 = 2$ $\therefore f'(2) = 10$ $x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 2 - \frac{f(2)}{f'(2)} = 2.1$ $x_2 = 2.1 - \frac{f(2.1)}{f'(2.1)} = 2.095$ $x_3 = 2.095 - \frac{f(2.095)}{f'(2.095)} = 2.095$ $x_4 = 2.095 - \frac{f(2.095)}{f'(2.095)} = 2.095$
15	Find a real root of the equation $x^3 + 4x - 9 = 0$ in the interval $(1,2)$ by using Bisection method. (only one iteration) Let $f(x) = x^3 + 4x - 9$ $f(1) = -4$ $f(2) = 7$ $\therefore$ the root is in $(1,2)$ $x_1 = \frac{a+b}{2} = \frac{1+2}{2} = 1.5$

16	$10x+y+2z=13$, $3x+10y+z=14$, $2x+3y+10z=15$ $x=\frac{1}{10}(13-y-2z)$ $y=\frac{1}{10}(14-3x-z)$ $z=\frac{1}{10}(15-2x-3y)$ Starting with $x_0=y_0=z_0=0$ $x_1=1.3$ $y_1=1.01$ $z_1=0.937$ $x_2=1.012$ $y_2=1.003$ $z_2=0.997$
17	$f(x)=x^2+x-5$ $f(1)=-3<0$ $f(2)=1>0$ $f'(x)=2x+1$ Initial root x_0=2 $\therefore f'(2)=5$ $x_1=x_0-\frac{f(x_0)}{f'(x_0)}=2-\frac{f(2)}{f'(2)}=1.8$ $x_2=1.8-\frac{f(1.8)}{f'(1.8)}=1.7913$ $x_3=1.7913-\frac{f(1.7913)}{f'(1.7913)}=1.7912$ $x_4=1.7912-\frac{f(1.7912)}{f'(1.7912)}=1.7912$
18	

	Prove that the root of the equation $x^3 - x - 4 = 0$ lies between 0 and 2 Let $f(x) = x^3 - x - 4$ $f(0) = -4 < 0$ $f(2) = 2 > 0$ $\therefore$ root lies between 0 and 2
19	Solve the following equtions by using Jacobi's method $20x + y - 2z = 17$, $3x + 20y - z + 18 = 0$, $2x - 3y + 20z = 25$ $20x + y - 2z = 17$, $3x + 20y - z = -18$, $2x - 3y + 20z = 25$ $x = \frac{1}{20}(17 - y + 2z)$ $y = \frac{1}{20}(-18 - 3x + z)$ $z = \frac{1}{20}(25 - 2x + 3y)$ Starting with $x_0 = y_0 = z_0 = 0$ $x_1 = 0.85$ $y_1 = -0.9$ $z_1 = 1.25$ $x_2 = 1.02$ $y_2 = -0.965$ $z_2 = 1.03$ $x_3 = 1.001$ $y_3 = -1.002$ $z_3 = 1.003$

20	Find the approximate rootsof the equation $x^3 - x - 4 = 0$ by bisection method. Let $f(x) = x^3 - x - 4$ $f(1) = -4 < 0$ $f(2) = 2 > 0$ $\therefore$ root lies in $(1,2)$ $x_1 = \frac{a+b}{2} = \frac{1+2}{2} = 1.5$ $f(1.5) = -2.125 < 0$ $\therefore$ the root lies in (1.5,2) $x_2 = \frac{x_1+b}{2} = \frac{1.5+2}{2} = 1.75$ $f(x_2) = -0.39 < 0$ $\therefore$ the root lies in (1.75,2) $x_3 = \frac{x_2+b}{2} = \frac{1.75+2}{2} = 1.875$
21	F(x)=X^3-7 Initial root=2 F'(X)=3X^2 X_1 =1.91 X_2=1.91 X_3=1.91
22	Show that the roots of the equation $x^3 - 9x + 1 = 0$ lies between 2 and 3. Obtain the roots by Bisection method (3 iterations only) Let $f(x) = x^3 - 9x + 1$

	$f(2) = -9 < 0$ $f(3) = 1 > 0$ $\because$ root lies in $(2,3)$ $x_1 = \frac{a+b}{2} = \frac{2+3}{2} = 2.5$ $f(x_1) = -5.875 < 0$ the root lies in $(2.5, 3)$ $x_2 = \frac{a+x_1}{2} = \frac{2.5+3}{2} = 2.75$ $f(x_2) = -2.953 < 0$ the root lies in $(2.75, 3)$ $x_3 = \frac{a+x_2}{2} = \frac{2.75+3}{2} = 2.875$
23	Using Regula-Falsi method , find the root of $xe^x - 3 = 0$ (three iterations only) Let $f(x) = xe^x - 3$ $f(1) = -0.282 < 0$ $f(2) = 11.778 > 0$ $\therefore$ the root lies in $(1, 2)$ $x_1 = \frac{af(b) - bf(a)}{f(b) - f(a)} = \frac{1(11.778) - 2(-0.282)}{11.778 + 0.282} = 1.023$ $f(x_1) = -0.154 < 0$ the root lies in $(1.023, 2)$ $x_2 = \frac{1.023(11.778) - 2(-0.154)}{11.778 + 0.154} = 1.036$ $f(x_2) = -0.081 < 0$ the root lies in $(1.036, 2)$ $x_3 = \frac{1.036(11.778) - 2(-0.081)}{11.778 + 0.081} = 1.043$

24	Using bisection method , find the approximate root of $x^3 - 2x - 5 = 0$ in the interval $(2,3)$ (3 iterations only) Let $f(x) = x^3 - 2x - 5$ $f(2) = -1 < 0$ $f(3) = 16 > 0$ $\therefore$ root lies in $(2,3)$ $x_1 = \frac{a+b}{2} = \frac{2+3}{2} = 2.5$ $f(x_1) = 5.625 > 0$ the root lies in $(2, 2.5)$ $x_2 = \frac{2+2.5}{2} = 2.25$ $f(x_2) = 1.891 > 0$ the root lies in $(2, 2.25)$ $x_3 = \frac{2+2.25}{2} = 2.125$
25	Find the root of the equation using Newton-Raphson method $x^2 - 4x - 6 = 0$ near to 5.(three iterations only) Let $f(x) = x^2 - 4x - 6$ $f(5) = -1 < 0$ $f'(x) = 2x - 4$ $\therefore f'(5) = 6$ Initial root $x_0 = 5$ $x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 5 - \frac{f(5)}{f'(5)} = 5.167$ $x_2 = 5.167 - \frac{f(5.167)}{f'(5.167)} = 5.162$ $x_2 = 5.162 - \frac{f(5.162)}{f'(5.162)} = 5.162$

26	Solve by Jacobi's method upto 3 iterations only: $30x + y + z = 32$, $x + 30y + z = 32$, $x + y + 30z = 32$ $x = \frac{1}{30}(32 - y - z)$ $y = \frac{1}{30}(32 - x - z)$ $z = \frac{1}{30}(32 - x - y)$ Starting with $x_0 = y_0 = z_0 = 0$ $x_1 = 1.067$ $y_1 = 1.067$ $z_1 = 1.067$ $x_2 = 0.996$ $y_2 = 0.996$ $z_2 = 0.996$ $x_3 = 1$ $y_3 = 1$ $z_3 = 1$

27	Solve by Gauss-Seidal method (3 iterations only) $6x + y + z = 105$, $4x + 8y + 3z = 155$, $5x + 4y - 10z = 65$ $x = \frac{1}{6}(105 - y - z)$ $y = \frac{1}{8}(155 - 4x - 3z)$ $z = \frac{1}{-10}(65 - 5x - 4y)$ Starting with $y_0 = z_0 = 0$ $x_1 = 17.5$ $y_1 = 10.625$ $z_1 = 6.5$ $x_2 = 14.646$ $y_2 = 9.615$ $z_2 = 4.669$ $x_3 = 15.119$ $y_3 = 10.065$ $z_3 = 5.086$

28	Solve by Jacobi's method $4x + y + 2z = 12 \quad , \quad -x + 11y + 4z = 33 \quad , \quad 2x - 3y + 8z = 20$ (3 iterations only) $x = \frac{1}{4}(12 - y - 2z)$ $y = \frac{1}{11}(33 + x - 4z)$ $z = \frac{1}{8}(20 - 2x + 3y)$ Starting with $x_0 = y_0 = z_0 = 0$ $x_1 = 3$ $y_1 = 3$ $z_1 = 2.5$ $x_2 = 1$ $y_2 = 2.364$ $z_2 = 2.875$ $x_3 = 0.972$ $y_3 = 2.045$ $z_3 = 3.137$
29	Using Gauss seidal method find first iteration for system of equations: $8x + 2y + 3z = 30 \quad , \quad x - 9y + 2z = 1 \quad , \quad 2x + 3y + 6z = 31$ $x = \frac{30 - 2y - 3z}{8}$ $y = \frac{1 - x - 2z}{-9}$ $z = \frac{31 - 2x - 3y}{6}$ Initial approximations : $x_0 = y_0 = z_0 = 0$ $x = 3.75$ $y = 0.306$ $z = 3.764$

30	Show that the root of the eqaution $xe^x - 3 = 0$ lies in the interval(1, 2) Let $f(x) = xe^x - 3$ $f(1) = -0.282 < 0$ $f(2) = 11.778 > 0$ $\therefore$ root lies between 1 and 2
31	Using Bisection method find the approximate root of the equation $x^3 - 6x + 3 = 0$ (Perform three iterations) $x^3 - 6x + 3 = 0$ $f(x) = x^3 - 6x + 3$ $f(0) = 3 > 0$ $f(1) = -2 < 0$ root is in $(0,1)$ $\therefore x_1 = \frac{0+1}{2} = 0.5$ $\therefore f(0.5) = 0.125 > 0$ $\therefore$ root is in $(0.5,1)$ $\therefore x_2 = \frac{0.5+1}{2} = 0.75$ $\therefore f(0.75) = -1.078 < 0$ $\therefore$ root is in $(0.5, 0.75)$ $\therefore x_3 = \frac{0.75+0.5}{2} = 0.625$
32	Use Regula-Falsi method, to find approximate root of the equation $x^3 - x - 4 = 0$ (Three iterations) $x^3 - x - 4 = 0$

	Let $f(x)=x^3-x-4$ $f(1)=-4<0$ $f(2)=2>0$ $\therefore$ the root is in $(1,2)$ $x_1=\dfrac{af(b)-bf(a)}{f(b)-f(a)}=\dfrac{1(2)-2(-4)}{2-(-4)}=1.667$ $f(x_1)=-1.035<0$ $\therefore$ the root is in $(1.667,2)$ $x_2=\dfrac{1.667(2)-2(-1.035)}{2-(-1.035)}=1.781$ $f(x_2)=-0.132<0$ $\therefore$ the root is in $(1.781,2)$ $x_3=\dfrac{1.781(2)-2(-0.132)}{2-(-0.132)}=1.795$
33	Use the Newton-Raphson method to evaluate $\sqrt[3]{20}$ (three iterations) Let $x=\sqrt[3]{20}$ $\therefore x^3=20$ Let $f(x)=x^3-20$ $f(2)=-12<0$ $f(3)=7>0$ $f'(x)=3x^2$ Initial root $x_0=3$ $\therefore f'(3)=27$ $x_1=x_0-\dfrac{f(x_0)}{f'(x_0)}=3-\dfrac{f(3)}{f'(3)}=2.741$ $x_2=2.74-\dfrac{f(2.741)}{f'(2.741)}=2.715$ $x_3=2.71-\dfrac{f(2.715)}{f'(2.715)}=2.714$

34	Using Bisection method find the root of the equation x^3-4x-9 in the interval $(2,3)$(Perform three iterations) $x^3-4x-9=0$ $f(x)=x^3-4x-9$ $f(2)=-9<0$ $f(3)=6>0$ root is in $(2,3)$ $\therefore x_1=\frac{2+3}{2}=2.5$ $\therefore f(2.5)=-3.375<0$ $\therefore$ root is in $(2.5,3)$ $\therefore x_2=\frac{2.5+3}{2}=2.75$ $\therefore f(2.75)=0.797>0$ $\therefore$ root is in $(2.75,2.5)$ $\therefore x_3=\frac{2.75+2.5}{2}=2.625$

35	Using Jacobi's method solve the system of equations: (Perform three iterations) $10x+2y+z=9$, $2x+20y-2z=-44$, $-2x+3y+10z=22$ $x=\frac{1}{10}(9-2y-z)$ $y=\frac{1}{20}(-44-2x+2z)$ $z=\frac{1}{10}(22+2x-3y)$ Starting with $x_0=y_0=z_0=0$ $x_1=0.9$ $y_1=-2.2$ $z_1=2.2$ $x_2=1.12$ $y_2=-2.07$ $z_2=3.04$ $x_3=1.01$ $y_3=-2.008$ $z_3=3.045$
36	Using Gauss-seidal method solve the system of equations: $5x-y=9$, $x-5y+z=-4$, $y-5z=15$ (Perform three iterations) $x=\frac{1}{5}(9+y)$ $y=\frac{1}{5}(4+x+z)$ $z=\frac{1}{-5}(15-y)$ Starting with $x_0=y_0=z_0=0$ $x_1=1.8$ $y_1=1.16$ $z_1=-2.768$

	$x_2 = 2.032$ $y_2 = 0.653$ $z_2 = -2.869$ $x_3 = 1.931$ $y_3 = 0.612$ $z_3 = -2.878$
37	Using Jacobi's method solve the system of equations: $2x+3y-4z=1$, $5x+9y+3z=17$, $8x-2y-z=5$ (Perform three iterations) $x=\frac{1}{8}(5+2y+z)$ $y=\frac{1}{9}(17-5x-3z)$ $z=-\frac{1}{4}(1-2x-3y)$ Starting with $x_0 = y_0 = z_0 = 0$ $x_1 = 0.625$ $y_1 = 1.889$ $z_1 = -0.25$ $x_2 = 1.066$ $y_2 = 1.625$ $z_2 = 1.479$ $x_3 = 1.216$ $y_3 = 0.804$ $z_3 = 1.502$

38	Show that there exist a root of the equation $x^3-4x+1=0$ in the interval $(1,2)$. Let $f(x)=x^3-4x+1$ $f(1)=-2<0$ $f(2)=1>0$ $\therefore$ root is in $(1,2)$
39	Find by Jacobi's method , the first iteration only, for the following equation $5x-y=9$, $x-5y+z=-4$, $y-5z=6$ $x=\frac{9+y}{5}$ $y=\frac{-4-x-z}{-5}$ $z=\frac{6-y}{-5}$ Initial approximations : $x_0=y_0=z_0=0$ $x=\frac{9}{5}$=1.8 $y=\frac{4}{5}$=0.8 $z=\frac{6}{-5}=-1.2$

40	Using false position method, find the root of the equation $x^2+x-3=0$ in the interval $(1,2)$ by performing three iterations. Let $f(x)=x^2+x-3$ $f(1)=-1<0$ $f(2)=3>0$ $\therefore$ the root is in $(1,2)$ $x_1=\dfrac{af(b)-bf(a)}{f(b)-f(a)}$ $=\dfrac{1(3)-2(-1)}{3-(-1)}=1.25$ $f(x_1)=-0.188<0$ $\therefore$ the root is in $(1.25,2)$ $x_2=\dfrac{1.25(3)-2(-0.188)}{3+0.188}=1.294$ $f(x_2)=-0.032<0$ $\therefore$ the root is in $(1.294,2)$ $x_3=\dfrac{1.294(3)-2(-0.032)}{3+0.032}=1.301$
41	Solve the equation $x^3-x-1=0$ using Newton-Raphson method taking initial root '1' [Three iterations only] Let $f(x)=x^3-x-1$ $f(1)=-1<0$ $f(2)=5>0$ $f'(x)=3x^2-1$ Initial root $x_0=1$ $\therefore f'(1)=2$ $x_1=x_0-\dfrac{f(x_0)}{f'(x_0)}=1-\dfrac{f(1)}{f'(1)}=1.5$ $x_2=1.5-\dfrac{f(1.5)}{f'(1.5)}=1.348$ $x_3=1.348-\dfrac{f(1.348)}{f'(1.348)}=1.325$

42	Find the root of $e^{-x}-x=0$ by bisection method (up to three iterations.) Let $f(x)=e^{-x}-x$ $\therefore f(0)=e^{-0}-0=1$ $f(1)=e^{-1}-1=-0.632$ $\therefore$ root is in $(0,1)$ $\therefore x_1=\dfrac{0+1}{2}=0.5$ $\therefore f(0.5)=0.107$ $\therefore$ root is in $(0.5,1)$ $\therefore x_2=\dfrac{0.5+1}{2}=0.75$ $\therefore f(0.75)=-0.278$ $\therefore$ root is in $(0.5,0.75)$ $\therefore x_3=\dfrac{0.5+0.75}{2}=0.625$
43	Solve the following equtions by Jacobi's method (take three iterations.) $20x+y-2z=17$, $2x-3y+20z=25$, $3x+20y-z=18$ $20x+y-2z=17$,$3x+20y-z=18$, $2x-3y+20z=25$ $x=\dfrac{1}{20}(17-y+2z)$ $y=\dfrac{1}{20}(18-3x+z)$

	$z=\frac{1}{20}(25-2x+3y)$ Starting with $x_0=y_0=z_0=0$ $x_1=0.85$ $y_1=0.9$ $z_1=1.25$ $x_2=0.93$ $y_2=0.835$ $z_2=1.3$ $x_3=0.938$ $y_3=0.826$ $z_3=1.282$
44	Solve the equations by Gauss Seidal method(up to three iterations.) $8x+2y+3z=30$, $x-9y+2z=1$, $2x+3y+6z=31$ $8x+2y+3z=30$, $x-9y+2z=1$, $2x+3y+6z=31$ $\therefore x=\frac{1}{8}(30-2y-3z)$ $y=\frac{1}{-9}(1-x-2z)$ $z=\frac{1}{6}(31-2x-3y)$ Starting with $x_0=y_0=z_0=0$ $x_1=3.75$ $y_1=0.306$ $z_1=3.764$ $x_2=2.262$ $y_2=0.977$ $z_2=3.924$

	$x_3 = 2.034$ $y_3 = 0.987$ $z_3 = 3.996$
45	With the following system of equations: $3x+2y=4.5,\ 2x+3y-z=5,\ -y+2z=0.52$ Set up the Gauss-Seidal iterations scheme for solution.Iterate two times using initial approximations$x_0=0.4, y_0=1.6, z_0=0.4$ $3x+2y=4.5,\ 2x+3y-z=5,\ -y+2z=0.52$ $\therefore x=\frac{1}{3}(4.5-2y)$ $y=\frac{1}{3}(5-2x+z)$ $z=\frac{1}{2}(0.52+y)$ Starting with $x_0=0.4, y_0=1.6, z_0=0.4$ $x_1=0.433$ $y_1=1.511$ $z_1=1.016$ $x_2=0.493$ $y_2=1.677$ $z_2=1.099$
46	Show that there exist a root of the equation $x^3+2x^2-8=0$ between 1 and 2. Let $f(x)=x^3+2x^2-8$ $f(1)=-5<0$ $f(2)=8>0$ $\therefore$ root lies between 1 and 2

47	Solve the f1ollowing equations by using Gauss-Seidal method (only first iteration) $10x+2y+z=9; x+10y-z=-22; -2x+3y+10z=22$ $x=\frac{9-2y-z}{10}$ $y=\frac{-22-x+z}{10}$ $z=\frac{22+2x-3y}{10}$ Initial approximations : $x_0=y_0=z_0=0$ $x_1=0.9$, $y_1=-2.29$, $z_1=3.067$
48	Using Regula-Falsi method, Find approximate root of $x^3-9x+1=0$ (Three iterations only) Let $f(x)=x^3-9x+1$ $f(2)=-9<0$ $f(3)=1>0$ $\therefore$ the root lies in $(2,3)$ $x_1=\frac{af(b)-bf(a)}{f(b)-f(a)}=\frac{2(1)-3(-9)}{1+9}=2.9$ $f(x_1)=-0.711<0$ $\therefore$ root lies in $(2.9,3)$ $x_2=\frac{2.9(1)-3(-0.711)}{1+0.711}=2.942$ $f(x_2)=-0.014<0$ the root lies in $(2.942,3)$ $x_3=\frac{2.942(1)-3(-0.014)}{1+0.014}=2.943$

49	Solve by Newton-Raphson method $x^3+2x-20=0$ (Three iteraions only) Let. $f(x)=x^3+2x-20$ $f(2)=-8<0$ $f(3)=13>0$ $f'(x)=3x^2+2$ $\therefore f'(2)=14$ $\therefore$ Initial root $x_0=2$ $x_1=x_0-\frac{f(x_0)}{f'(x_0)}=2.571$ $x_2=x_1-\frac{f(x_1)}{f'(x_1)}=2.473$ $x_3=x_2-\frac{f(x_2)}{f'(x_2)}=2.4695$

50	$10x+y+2z=13$, $3x+10y+z=14$, $2x+3y+10z=15$ $x=\frac{1}{10}(13-y-2z)$ $y=\frac{1}{10}(14-3x-z)$ $z=\frac{1}{10}(15-2x-3y)$ Starting with $x_0=y_0=z_0=0$ $x_1=1.3$ $y_1=1.01$ $z_1=0.937$ $x_2=1.012$ $y_2=1.003$ $z_2=0.997$
51	From the following system of Equations, $3x+2y=4.5,\ 2x+3y-z=5,\ -y+2z=0.52$ Find one iteration only using Gauss-seidal method. $3x+2y=4.5,\ 2x+3y-z=5,\ -y+2z=0.52$ $x=\frac{4.5-2y}{3}$ $y=\frac{5-2x+z}{3}$ $z=\frac{0.52+y}{2}$ $x_1=1.5,\ y_1=0.667,\ z_1=0.594$

52	Find approximate root of the equation $x.\log_e x = 1.2$ by using bisection method. $x.\log_e x = 1.2$ $x.\log_e x - 1.2 = 0$ $f(x) = x.\log_e x - 1.2$ $f(1) = -1.2 < 0$ $f(2) = 0.186 > 0$ $\therefore$ the root is in $(1, 2)$ $x_1 = \frac{a+b}{2} = \frac{1+2}{2} = 1.5$ $f(1.5) = -0.592 < 0$ $\therefore$ the root is in (1.5,2) $x_2 = \frac{x_1+b}{2} = \frac{1.5+2}{2} = 1.75$ $f(1.75) = -0.221 < 0$ the root is in $(1.75, 2)$ $x_3 = \frac{x_2+b}{2} = \frac{1.75+2}{2} = 1.875$
53	Use Newton-Raphson method to find root of equation $x^2 + x - 3 = 0$ (up to three iterations) Let $f(x) = x^2 + x - 3$ $f(1) = -1 < 0$ $f(2) = 3 > 0$ $f'(x) = 2x + 1$ Initial root $x_0 = 1$ $\therefore f'(1) = 3$ $x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 1 - \frac{f(1)}{f'(1)} = 1.333$ $x_2 = 1.333 - \frac{f(1.333)}{f'(1.333)} = 1.303$ $x_3 = 1.303 - \frac{f(1.303)}{f'(1.303)} = 1.303$

54	Solve the following equations using Gauss-Seidal method. $10x+y+z=12$, $x+10y+z=12$, $x+y+10z=12$ $x=\frac{1}{10}(12-y-z)$ $y=\frac{1}{10}(12-x-z)$ $z=\frac{1}{10}(12-x-y)$ Starting with $x_0=y_0=z_0=0$ $x_1=1.2$ $y_1=1.08$ $z_1=0.972$ $x_2=0.995$ $y_2=1.003$ $z_2=1$ $x_3=1$ $y_3=1$ $z_3=1$
55	Show that root of equation $x^2+x-3=0$ lies between 2 and 3. $f(x)=x^2+x-3$ $f(2)=2^2+2-3=3>0$ $f(3)=3^2+3-3=9>0$ root not in 2 and 3

56	With the following system of equations $5x - y = 9,\ 5y - z = 6,\ x + 5z = -3$ Set up Gauss Seidal iteration scheme for the solution. Iterate two times using initial approximations $x_0 = 1.8, y_0 = 1.2, z_0 = -0.96$ $5x - y = 9,\ 5y - z = 6,\ x + 5z = -3$ $\therefore x = \frac{1}{5}(9 + y)$ $y = \frac{1}{5}(6 + z)$ $z = \frac{1}{5}(-3 - x)$ Starting with $x_0 = 1.8, y_0 = 1.2, z_0 = -0.96$ $x_1 = 2.04$ $y_1 = 1.008$ $z_1 = -1.008$ $x_2 = 2.002$ $y_2 = 0.998$ $z_2 = -1.000$
57	N=67 x_0=8 Ist iteration- x_1=8.81875 IInd iteration- x_2=8.18535305 Third iteration- x_3=8.18535277
58	N=102 X_0=10 Ist iteration- X_1=10.1 IInd iteration- X_2= 10.0995049 Third iteration- X_3=10.09950
59	N=85 x_0=9

	Ist iteration- x_1=9.222222 IInd iteration- x_2=9.21954484 Third iteration- x_3=9.21954445
60	$N = 26$ $x_0 = 5$ $Ist\ iteration -$ $x_1 = 5.09999$ $IInd\ iteration -$ $x_2 = 5.0990196$ $Third\ iteration -$ $x_3 = 5.0990195$

Thank You

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