Unit III	Differential Equation	Marks -12

Q. N.	Solution
1	$\frac{d^2y}{dx^2}=\sqrt{1+\frac{dy}{dx}}$ $\frac{d^2y}{dx^2}=\sqrt{1+\frac{dy}{dx}}$ Squaring both sides, we get $\left(\frac{d^2y}{dx^2}\right)^2=1+\frac{dy}{dx}$ $\therefore$ Order $=2$ Degree $=2$
2	Find the order and degree of the differential equation $\frac{d^2y}{dx^2}=\left(y+\frac{dy}{dx}\right)^{3/2}$ $\frac{d^2y}{dx^2}=\left(y+\frac{dy}{dx}\right)^{3/2}$ squaring $\left(\frac{d^2y}{dx^2}\right)^2=\left(y+\frac{dy}{dx}\right)^3$ Order of D.E. $=2$ Degree of D.E. $=2$
3	$Given, \frac{d^2y}{dx^2}=\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}$ $\therefore \left(\frac{d^2y}{dx^2}\right)^2=\left[1+\left(\frac{dy}{dx}\right)^2\right]^3$ $Order=2, Degree=2$

4	Find order and degree of the differential equation $\frac{d^2y}{dx^2}=\sqrt[4]{y+\left(\frac{dy}{dx}\right)^2}$ $\frac{d^2y}{dx^2}=\sqrt[4]{y+\left(\frac{dy}{dx}\right)^2}$ $\left(\frac{d^2y}{dx^2}\right)^4=y+\left(\frac{dy}{dx}\right)^2$ $\therefore$ Order = 2 Degree = 4
5	$Given, \frac{d^2y}{dx^2}=\sqrt[4]{1+\left(\frac{dy}{dx}\right)^2}$ $\therefore \left(\frac{d^2y}{dx^2}\right)^4=1+\left(\frac{dy}{dx}\right)^2$ $Order=2, Degree=4$
6	Find the order and degree of D.E. $\sqrt{\frac{d^2y}{dx^2}}-\frac{dy}{dx}-xy^2=0$ $\sqrt{\frac{d^2y}{dx^2}}=\frac{dy}{dx}+xy^2$ $\therefore \frac{d^2y}{dx^2}=\left(\frac{dy}{dx}+xy^2\right)^2$ $\therefore$ Order = 2 Degree = 1
7	$Given, \sqrt{\frac{dy}{dx}}=\sqrt[3]{\frac{d^2y}{dx^2}}$ $\therefore \left(\frac{dy}{dx}\right)^3=\left(\frac{d^2y}{dx^2}\right)^2$ $Order=2, Degree=2$
8	Find the order and degree of the differential equation. $\left(\frac{d^2y}{dx^2}\right)^{2/3}=\sqrt{y+\frac{dy}{dx}}$ $\left(\frac{d^2y}{dx^2}\right)^{2/3}=\sqrt{y+\frac{dy}{dx}}$ order = 2 $\left(\frac{d^2y}{dx^2}\right)^2=\left(y+\frac{dy}{dx}\right)^3$ degree = 2

9	Determine the order and degree of $x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}=my$ Order=2 Degree=1
10	$Given, \frac{d^2y}{dx^2} = 4\sqrt{1+\left(\frac{dy}{dx}\right)^2}$ $\therefore \left(\frac{d^2y}{dx^2}\right)^2 = 16\left(1+\left(\frac{dy}{dx}\right)^2\right)$ $Order = 2, Degree = 2$
11	Find the order and degree of the following equation: $\frac{d^2y}{dx^2}+\sqrt{1+\frac{dy}{dx}}=0$ Order = 2 For degree, $\left(\frac{d^2y}{dx^2}\right)^2=1+\frac{dy}{dx}$ $\therefore$ Degree = 2
12	Find the order and degree of the differential equation $\frac{d^2y}{dx^2}=\sqrt{y-\frac{dy}{dx}}$ $\frac{d^2y}{dx^2}=\sqrt{y-\frac{dy}{dx}}$ $\therefore$ Order = 2 Squaring both sides, we get $\left(\frac{d^2y}{dx^2}\right)^2=y-\frac{dy}{dx}$ Degree = 2
13	Form the differential equation of $y=a\sin x+b\cos x$ $y=a\sin x+b\cos x$ $\therefore \frac{dy}{dx}=a\cos x-b\sin x$ $\therefore \frac{d^2y}{dx^2}=-a\sin x-b\cos x$ $\therefore \frac{d^2y}{dx^2}=-(a\sin x+b\cos x)$ $\frac{d^2y}{dx^2}=-y$ $\frac{d^2y}{dx^2}+y=0$

14	Form a differential equation by eliminating arbitrary constant If $y = A\sin x + B\cos x$ $y = A\sin x + B\cos x$ $\therefore \frac{dy}{dx} = A\cos x - B\sin x$ $\therefore \frac{d^2y}{dx^2} = A(-\sin x) - B\cos x$ $= -(A\sin x + B\cos x) = -y$ $\therefore \frac{d^2y}{dx^2} + y = 0$
15	From the differential equation by eliminating the arbitray constant if $y = A\cos x + B\sin x.$ $y = A\cos x + B\sin x.$ $\frac{dy}{dx} = -A\sin x + B\cos x$ $\frac{d^2y}{dx^2} = -A\cos x - B\sin x$ $= -(A\cos x + B\sin x)$ $= -y$ $\frac{d^2y}{dx^2} + y = 0$
16	$y = A.e^x + B.e^{-x}$ $\therefore \frac{dy}{dx} = A.e^x - B.e^{-x}$ $\therefore \frac{d^2y}{dx^2} = A.e^x + B.e^{-x}$ $\therefore \frac{d^2y}{dx^2} = y$ $\therefore \frac{d^2y}{dx^2} - y = 0$
17	Form the D.E. by eliminating the arbitrary constants if $y = A\cos 3x + B\sin 3x$ $y = A\cos 3x + B\sin 3x$ $\therefore \frac{dy}{dx} = -3A\sin 3x + 3B\cos 3x$ $\therefore \frac{d^2y}{dx^2} = -9A\cos 3x - 9B\sin 3x$ $\therefore \frac{d^2y}{dx^2} = -9(A\cos 3x + B\sin 3x)$ $\frac{d^2y}{dx^2} = -9y$ $\frac{d^2y}{dx^2} + 9y = 0$
18	$y = a\ cos4x + b\ sin4x.$ $\therefore \frac{dy}{dx} = -4asin4x + 4bcos4x$ $\therefore \frac{d^2y}{dx^2} = -16acos4x - 16bsin4x$ $\therefore \frac{d^2y}{dx^2} = -16(acos4x + bsin4x)$

	$\therefore \frac{d^2y}{dx^2} = -16y$
19	Verify that $y = \log x$ is a solution of differential equation $x\frac{d^2y}{dx^2} + \frac{dy}{dx} = 0$ $y = \log x$ $\therefore \frac{dy}{dx} = \frac{1}{x}$ $\therefore x\frac{dy}{dx} = 1$ $\therefore x\frac{d^2y}{dx^2} + \frac{dy}{dx} = 0$ **<u>OR</u>** $y = \log x$ $\therefore \frac{dy}{dx} = \frac{1}{x}$ $\therefore \frac{d^2y}{dx^2} = -\frac{1}{x^2}$ $L.H.S. = x\frac{d^2y}{dx^2} + \frac{dy}{dx} = x\left(-\frac{1}{x^2}\right) + \frac{1}{x}$ $= -\frac{1}{x} + \frac{1}{x}$ $= 0 = R.H.S.$
20	Form the diffrential equation by eliminating the arbitrary constants if $y = a\cos(\log x) + b\sin(\log x)$ $y = a\cos(\log x) + b\sin(\log x)$ $\therefore \frac{dy}{dx} = -a\sin(\log x)\frac{1}{x} + b\cos(\log x)\frac{1}{x}$ $\therefore x\frac{dy}{dx} = -a\sin(\log x) + b\cos(\log x)$ $\therefore x\frac{d^2y}{dx^2} + \frac{dy}{dx} = -a\cos(\log x)\frac{1}{x} - b\sin(\log x)\frac{1}{x}$ $\therefore x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} = -(a\cos(\log x) + b\sin(\log x))$ $\therefore x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} = -y$ $\therefore x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$
21	Form the differential equation by eliminating the arbitrary constants if $y^2 = 4ax$ $y^2 = 4ax$ ------------(1) $2y\frac{dy}{dx} = 4a$ ------------(2) Put (2) in (1) $\therefore y^2 = 2y\frac{dy}{dx}x$ $\therefore y = 2x\frac{dy}{dx}$ $\therefore 2x\frac{dy}{dx} - y = 0$

22	Show that $y = A\sin mx + B\cos mx$ is a solution of differential equation $\frac{d^2y}{dx^2} + m^2y = 0$ $y = A\sin mx + B\cos mx$ $\frac{dy}{dx} = mA\cos mx - mB\sin mx$ $\frac{d^2y}{dx^2} = -m^2A\sin mx - m^2B\cos mx$ $\frac{d^2y}{dx^2} = -m^2\left(A\sin mx + B\cos mx\right)$ $\frac{d^2y}{dx^2} = -m^2y$ $\frac{d^2y}{dx^2} + m^2y = 0$
23	Solve: $\frac{dy}{dx} = e^{2x-y} + x^2e^{-y}$ $\frac{dy}{dx} = e^{2x-y} + x^2e^{-y}$ $\therefore \frac{dy}{dx} = \left(e^{2x}\cdot e^{-y} + x^2e^{-y}\right)$ $\therefore \frac{dy}{dx} = e^{-y}\left(e^{2x} + x^2\right)$ $\therefore \frac{dy}{e^{-y}} = \left(e^{2x} + x^2\right)dx$ $\therefore \int e^y dy = \int\left(e^{2x} + x^2\right)dx$ $\therefore e^y = \frac{e^{2x}}{2} + \frac{x^3}{3} + c$
24	Solve : $e^{x+y}dx + e^{2y-x}dy = 0$ $e^{x+y}dx + e^{2y-x}dy = 0$ $\therefore e^xe^ydx + e^{2y}e^{-x}dy = 0$ $\frac{e^x}{e^{-x}}dx = -\frac{e^{2y}}{e^y}dy$ $e^{2x}dx = -e^ydy$ $\int e^{2x}dx = -\int e^ydy$ $\frac{e^{2x}}{2} = -e^y + c$

25	Solve $\left(1+x^3\right)dy-x^2ydx=0$ $\because \left(1+x^3\right)dy-x^2ydx=0$ $\therefore \left(1+x^3\right)dy=x^2ydx$ $\therefore \frac{dy}{y}=\frac{x^2dx}{1+x^3}$ $\therefore$ Solution is, $\int\frac{dy}{y}=\int\frac{x^2}{1+x^3}dx$ $=\frac{1}{3}\int\frac{3x^2}{1+x^3}dx$ $\therefore \log y=\frac{1}{3}\log\left(1+x^3\right)+c$
26	Solve $\left(1+x^2\right)dy-x^2.ydx=0$ $\left(1+x^2\right)dy-x^2.ydx=0$ $\left(1+x^2\right)dy=x^2.ydx$ $\frac{dy}{y}=\frac{x^2dx}{1+x^2}$ $\int\frac{dy}{y}=\int\frac{x^2dx}{1+x^2}$ $\int\frac{dy}{y}=\int\frac{1+x^2-1dx}{1+x^2}$ $\int\frac{dy}{y}=\int\left[1-\frac{1}{1+x^2}\right]dx$ $\log y=x-\tan^{-1}x+c$
27	Solve $(x+1)\frac{dy}{dx}-y=e^x(x+1)^2$ $(x+1)\frac{dy}{dx}-y=e^x(x+1)^2$ $\therefore \frac{dy}{dx}-\frac{y}{x+1}=e^x(x+1)$ Comparing with $\frac{dy}{dx}+Py=Q$ $\therefore P=\frac{-1}{x+1},\quad Q=e^x(x+1)$ Integrating Factor $=e^{\int Pdx}=e^{\int\frac{-1}{x+1}dx}$ $=e^{-\log(x+1)}$ $=\frac{1}{x+1}$ $\therefore$ Solution is, $y\cdot I.F.=\int Q\cdot I.F.dx+c$ $y\cdot\frac{1}{x+1}=\int e^x(x+1)\cdot\frac{1}{x+1}dx+c$ $\therefore \frac{y}{x+1}=\int e^xdx+c$ $\therefore \frac{y}{x+1}=e^x+c$

28	Solve : $x\left(1+y^2\right)dx+y\left(1+x^2\right)dy=0$ $x\left(1+y^2\right)dx+y\left(1+x^2\right)dy=0$ $\therefore \frac{x}{1+x^2}dx=-\frac{y}{1+y^2}dy$ $\therefore \int\frac{x}{1+x^2}dx=-\int\frac{y}{1+y^2}dx$ $\therefore \frac{1}{2}\log\left(1+x^2\right)=-\frac{1}{2}\log\left(1+y^2\right)+c$ $\therefore \log\left(1+x^2\right)=-\log\left(1+y^2\right)+C$
29	Solve the D.E. $x\sqrt{1-y^2}dx+y\sqrt{1-x^2}dy=0$ $x\sqrt{1-y^2}dx+y\sqrt{1-x^2}dy=0$ $\therefore x\sqrt{1-y^2}dx=-y\sqrt{1-x^2}dy$ $\therefore \frac{xdx}{\sqrt{1-x^2}}=\frac{-ydy}{\sqrt{1-y^2}}$ $\therefore \int\frac{xdx}{\sqrt{1-x^2}}=\int\frac{-ydy}{\sqrt{1-y^2}}$ $\therefore \frac{1}{-2}\int\frac{-2xdx}{\sqrt{1-x^2}}=\frac{1}{2}\int\frac{-2ydy}{\sqrt{1-y^2}}$ $\therefore \frac{1}{-2}2\sqrt{1-x^2}=\frac{1}{2}2\sqrt{1-y^2}+c$ $\therefore -\sqrt{1-x^2}-\sqrt{1-y^2}=c$ or $\sqrt{1-x^2}+\sqrt{1-y^2}=c$
30	Solve the differential equation: $\frac{dy}{dx}+y\tan x=\cos^2 x$ $\frac{dy}{dx}+y\tan x=\cos^2 x$ Comparing with $\frac{dy}{dx}+Py=Q$ $\therefore P=\tan x$ and $Q=\cos^2 x$ $IF=e^{\int\tan xdx}=e^{\log(\sec x)}=\sec x$ $\therefore y\cdot IF=\int Q\cdot IFdx+c$ $y\cdot\sec x=\int\cos^2 x\sec xdx+c$ $y\cdot\sec x=\int\cos xdx+c$ $y\cdot\sec x=\sin x+c$
31	Solve $\frac{dy}{dx}+y\cot x=\cos ecx$ $\frac{dy}{dx}+y\cot x=\cos ecx$ Comparing with $\frac{dy}{dx}+Py=Q$ $P=\cot x$, $Q=\cos ecx$ Integrating factor $IF=e^{\int\cot xdx}=e^{\log(\sin x)}=\sin x$ $y.IF=\int Q.IFdx+c$ $\therefore y\sin x=\int\cos ecx.\sin x\ dx$ $\therefore y\sin x=\int 1\ dx$ $\therefore y\sin x=x+c$

32	$\frac{dy}{dx}+y\cot x=\cos x$ $\therefore$ Comparing with $\frac{dy}{dx}+Py=Q$ $P=\cot x$, $Q=\cos x$ Integrating factor $IF=e^{\int \cot x dx}=e^{\log(\sin x)}=\sin x$ $y.IF=\int Q.IF dx+c$ $\therefore y\sin x=\int \cos x.\sin x\, dx$ Put $\sin x=t$ $\therefore \cos x\, dx=dt$ $=\int t\, dt$ $=\frac{t^2}{2}+c$ $\therefore y\sin x=\frac{(\sin x)^2}{2}+c$
33	Solve: $x\frac{dy}{dx}-y=x^2$ $x\frac{dy}{dx}-y=x^2$ Divide by x $\frac{dy}{dx}-\frac{y}{x}=x$ $\therefore$ Comparing with $\frac{dy}{dx}+Py=Q$ $P=\frac{-1}{x}$, $Q=x$ Integrating factor $IF=e^{\int \frac{-1}{x}dx}=e^{-\log x}=\frac{1}{x}$ $y.IF=\int Q.IF dx+c$ $y\frac{1}{x}=\int x.\frac{1}{x}dx$ $\frac{y}{x}=\int 1dx$ $\frac{y}{x}=x+c$
34	

	Solve D.E. $x\cdot\frac{dy}{dx}+y=x^3$ $x\cdot\frac{dy}{dx}+y=x^3$ $\therefore \frac{dy}{dx}+\frac{y}{x}=x^2$ Comparing with $\frac{dy}{dx}+Py=Q$ $\therefore P=\frac{1}{x},\ Q=x^2$ $\therefore$ Integrating Factor $=e^{\int P dx}$ $=e^{\int \frac{1}{x} dx}$ $=e^{\log x}$ $=x$ $\therefore$ Solution is, $y\cdot \text{I.F.}=\int Q\cdot \text{I.F.}\,dx+c$ $\therefore y\cdot x=\int x^2\cdot x dx+c$ $\therefore xy=\int x^3 dx+c$ $\therefore xy=\frac{x^4}{4}+c$

35	Solve $x\log x\frac{dy}{dx}+y=2\log x$ $x\log x\frac{dy}{dx}+y=2\log x$ $\frac{dy}{dx}+\frac{y}{x\log x}=\frac{2\log x}{x\log x}$ $\frac{dy}{dx}+\frac{y}{x\log x}=\frac{2}{x}$ $I.F=e^{\int\frac{1}{x\log x}dx}$ $=e^{\log(\log x)}=\log x$ Solution is, $y.\log x=\int\frac{2}{x}.\log x dx$ Let $I_1=\int\frac{2}{x}.\log x dx$ Put $\log x=t$ $\frac{1}{x}dx=dt$ $\therefore I_1=2\int t dt$ $=2\frac{t^2}{2}+c$ $=(\log x)^2+c$ $y.\log x=(\log x)^2+c$
36	Solve : $\frac{dy}{dx}+\frac{y}{x}=x^2$ $\frac{dy}{dx}+\frac{y}{x}=x^2$ Comparing with $\frac{dy}{dx}+Py=Q$ $\therefore P=\frac{1}{x}$ and $Q=x^2$ $IF=e^{\int\frac{1}{x}dx}=e^{\log x}=x$ $\therefore$ Solution is $y\cdot IF=\int Q\cdot IF dx+c$ $y\cdot x=\int x^2 x dx+c$ $xy=\int x^3 dx+c$ $xy=\frac{x^4}{4}+c$
37	Solve $(x^3+y^3)\frac{dy}{dx}=x^2y$ $\therefore\frac{dy}{dx}=\frac{x^2y}{x^3+y^3}$

	Put $y = vx$ $\quad \therefore \frac{dy}{dx} = v + x\frac{dv}{dx}$ $\therefore v + x\frac{dv}{dx} = \frac{x^2 vx}{x^3 + (vx)^3} = \frac{v}{1+v^3}$ $\therefore x\frac{dv}{dx} = \frac{v}{1+v^3} - v$ $\therefore x\frac{dv}{dx} = -\frac{v^4}{1+v^3}$ $\therefore \frac{1+v^3}{v^4}dv = -\frac{1}{x}dx$ $\therefore \int\left(\frac{1}{v^4}+\frac{1}{v}\right)dv = -\int\frac{1}{x}dx$ $\therefore \frac{v^{-3}}{-3} + \log v = -\log x + c$ $\therefore \frac{1}{-3v^3} + \log v = -\log x + c$ $\therefore \frac{x^3}{-3y^3} + \log\left(\frac{y}{x}\right) = -\log x + c$
38	Solve $\sec^2 x \tan y dx + \sec^2 y \tan x dy = 0$ $\sec^2 x \tan y dx + \sec^2 y \tan x dy = 0$ $\sec^2 x \tan y dx = -\sec^2 y \tan x dy$ $\frac{\sec^2 x}{\tan x}dx = -\frac{\sec^2 y}{\tan y}dy$ $\therefore$ Solution is, $\int\frac{\sec^2 x}{\tan x}dx = -\int\frac{\sec^2 y}{\tan y}dy$ $\log(\tan x) = -\log(\tan y) + c$
39	Solve $(1+x^2)dy - (1+y^2)dx = 0$ $(1+x^2)dy - (1+y^2)dx = 0$ $\therefore (1+x^2)dy = (1+y^2)dx \qquad \therefore \frac{dy}{1+y^2} = \frac{dx}{1+x^2}$ $\therefore \int\frac{dy}{1+y^2} = \int\frac{dx}{1+x^2}$ $\therefore \tan^{-1} y = \tan^{-1} x + c$

40	$x\frac{dy}{dx} - y = x^2$ $\therefore \frac{dy}{dx} - \frac{1}{x}y = x$ $\therefore P = -\frac{1}{x}$ and $Q = x$ $IF = e^{-\int \frac{1}{x}dx} = e^{-\log x} = \frac{1}{x}$
41	Solve the D.E. $x\frac{dy}{dx} - y = x^2 \cos^2 x$ $\frac{dy}{dx} - \frac{y}{x} = x\cos^2 x$ $\therefore P = -\frac{1}{x}$ and $Q = x\cos^2 x$ $IF = e^{-\int \frac{1}{x}dx} = e^{-\log x} = \frac{1}{x}$ $\therefore yIF = \int Q\,IF\,dx + c$ $y\frac{1}{x} = \int x\cos^2 x \frac{1}{x}dx + c$ $\frac{y}{x} = \int \cos^2 x dx + c$ $\frac{y}{x} = \frac{1}{2}\int (1 - \cos 2x)dx + c$ $\frac{y}{x} = \frac{1}{2}\left(x - \frac{\sin 2x}{2}\right) + c$
42	x (x + y) dy – y^2 dx = 0 $(x^2 + xy)dy - y^2dx = 0$ $\frac{dy}{dx} = \frac{y^2}{(x^2 + xy)}$ $Put\ y = vx, \quad \therefore \frac{dy}{dx} = v + x\frac{dv}{dx}$ $v + x\frac{dv}{dx} = \frac{v^2x^2}{x^2 + x^2v}$ $v + x\frac{dv}{dx} = \frac{v^2}{1+v}$ $x\frac{dv}{dx} = \frac{v^2}{1+v} - v$ $x\frac{dv}{dx} = \frac{-v}{1+v}$ $\frac{1+v}{-v}dv = \frac{1}{x}dx$ $(\frac{-1}{v} - 1)dv = \frac{1}{x}dx$

	$-\int \frac{1}{v} dv - \int 1\, dv = \int \frac{1}{x} dx$ $-logv - v = logx$ $-log\left(\frac{y}{x}\right) - \frac{y}{x} = logx$
43	Solve : $\frac{dy}{dx} = e^{3x-2y} + x^2 e^{-2y}$ $\frac{dy}{dx} = e^{3x} e^{-2y} + x^2 e^{-2y}$ $= e^{-2y}\left(e^{3x} + x^2\right)$ $\frac{dy}{e^{-2y}} = \left(e^{3x} + x^2\right) dx$ $e^{2y} dy = \left(e^{3x} + x^2\right) dx$ $\int e^{2y} dy = \int \left(e^{3x} + x^2\right) dx$ $\frac{e^{2y}}{2} = \frac{e^{3x}}{3} + \frac{x^3}{3} + c$
44	Solve the differential equation $\frac{dy}{dx} = e^x e^{-y} + x e^{-y}$ $\frac{dy}{dx} = e^x e^{-y} + x e^{-y}$ $\frac{dy}{dx} = e^{-y}\left(e^x + x\right) dx$ $\therefore e^y dy = \left(e^x + x\right) dx$ sol^n. is , $\int e^y dy = \int \left(e^x + x\right) dx$ $e^y = e^x + \frac{x^2}{2} + c$
45	Find integrating factor of $\left(1+x^2\right)\frac{dy}{dx} + y = e^{\tan^{-1}x}$ $\left(1+x^2\right)\frac{dy}{dx} + y = e^{\tan^{-1}x}$ $\therefore \frac{dy}{dx} + \frac{y}{1+x^2} = \frac{e^{\tan^{-1}x}}{1+x^2}$ Comparing with $\frac{dy}{dx} + Py = Q$ $\therefore P = \frac{1}{1+x^2}$ $\therefore$ Integrating factor $= e^{\int P dx}$ $= e^{\int \frac{1}{1+x^2} dx}$ $= e^{\tan^{-1}x}$

46	Solve $y^3.\sec^2 x dx + \left(3y^2 \tan x - \sec^2 y\right)dy = 0$ $y^3.\sec^2 x dx + \left(3y^2 \tan x - \sec^2 y\right)dy = 0$ Comparing with $\int M dx + \int N dy = c$ $M = y^3.\sec^2 x$ $\quad$ $N=3y^2 \tan x - \sec^2 y$ $\frac{\partial M}{\partial y} = 3y^2 \sec^2 x$ $\quad$ $\frac{\partial N}{\partial x} = 3y^2 \sec^2 x$ $\therefore \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ D.E. is exact $\therefore$ Solution is $\int y^3.\sec^2 x dx + \int -\sec^2 y dy = c$ $y^3 \tan x - \tan y = c$
47	Solve D.E. $\left(2xy + y^2\right)dx + \left(x^2 + 2xy + \sin y\right)dy = 0$ Let $M = 2xy + y^2$ $\quad$ $N = x^2 + 2xy + \sin y$ $\therefore \frac{\partial M}{\partial y} = 2x + 2y$ $\quad$ $\therefore \frac{\partial N}{\partial x} = 2x + 2y$ $\therefore \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ $\therefore$ D.E. exact Solution is $\int_{y-constant} M\ dx + \int_{terms\ not\ containing\ 'x'} N dy = c$ $\therefore \int_{y-constant} \left(2xy + y^2\right)\ dx + \int \sin y\ dy = c$ $\therefore x^2 y + xy^2 - \cos y = c$
48	Solve: $\left[y\left(1+\frac{1}{x}\right) + \cos y\right]dx + \left[x + \log x - x\sin y\right]dy = 0$ Let $M = y + \frac{y}{x} + \cos y$ $\quad , \quad$ $N = x + \log x - x\sin y$ $\therefore \frac{\partial M}{\partial y} = 1 + \frac{1}{x} - \sin y$ $\quad , \quad$ $\frac{\partial N}{\partial x} = 1 + \frac{1}{x} - \sin y$ $\therefore \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ $\therefore$ D.E. is exact Solution is, $\int_{y-constant} M\ dx + \int_{terms\ not\ containing\ 'x'} N dy = c$ $\therefore \int_{y-constant} \left(y + \frac{y}{x} + \cos y\right)\ dx + \int 0\ dy = c$ $\therefore yx + y\log x + x\cos y = c$

49	Solve $\left(3x^2+6xy^2\right)dx+\left(6x^2y+4y^2\right)dy=0$ $\left(3x^2+6xy^2\right)dx+\left(6x^2y+4y^2\right)dy=0$ Comparing with $Mdx+Ndy=0$ $\therefore M=3x^2+6xy^2, N=6x^2y+4y^2$ $\frac{\partial M}{\partial y}=12xy \quad , \frac{\partial N}{\partial x}=12xy$ $\because \frac{\partial M}{\partial y}=\frac{\partial N}{\partial x}$ $\therefore$ D.E. is an exact $\therefore$ Solution is, $\int_{y-\text{constant}} Mdx+\int_{\text{terms free from}'x'} Ndy=c$ $\therefore \int_{y-\text{constant}}\left(3x^2+6xy^2\right)dx+\int 4y^2dy=c$ $\therefore x^3+3x^2y^2+\frac{4}{3}y^3=c$
50	Solve $\left(2x^2+6xy-y^2\right)dx+\left(3x^2-2xy+y^2\right)dy=0$ $M=2x^2+6xy-y^2$ $\therefore \frac{\partial M}{\partial y}=6x-2y$ $N=3x^2-2xy+y^2$ $\therefore \frac{\partial N}{\partial x}=6x-2y$ $\therefore$ *the equation is exact.* $\int_{y\ \text{constant}} Mdx+\int_{\text{terms free from}\,x} Ndy=c$ $\int\left(2x^2+6xy-y^2\right)dx+\int y^2dy=c$ $\therefore 2.\frac{x^3}{3}+6y\cdot\frac{x^2}{2}-y^2x+\frac{y^3}{3}=c$ *or* $\frac{2}{3}x^3+3x^2y-y^2x+\frac{y^3}{3}=c$

Thank You

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