Unit II	Definite Integration	Marks - 12

Qs. No.	Solution
1	Evaluate: $\int x(x-1)^2 dx$ $\int x(x-1)^2 dx$ $= \int x(x^2-2x+1)\, dx$ $= \int (x^3-2x^2+x)\, dx$ $= \frac{x^4}{4} - \frac{2x^3}{3} + \frac{x^2}{2} + c$
2	Evaluate $\int \frac{1}{x^2+4}\, dx$ $\int \frac{1}{x^2+4}\, dx$ $= \int \frac{1}{x^2+(2)^2}\, dx$ $= \frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right) + c$
3	Let I = $\int \frac{1}{\sqrt{9-4x^2}}\, dx$ $\therefore I = \int \frac{1}{\sqrt{4(\frac{9}{4}-x^2)}} dx$ $\therefore I = \frac{1}{\sqrt{4}} \int \frac{1}{\sqrt{\left(\frac{3}{2}\right)^2 - x^2}} dx$ $\therefore I = \frac{1}{2}\sin^{-1}\frac{x}{3/2} + c$

4	$Let\ I = \int_0^{\pi/2} sinx.cosx\ \ dx$ $Put\ sinx = t$ $cosx\ dx = dt$ $At\ x = 0, \quad sin0 = t, \quad \therefore 0 = t$ $At\ x = \pi/2, \quad sin\,\pi/2 = t, \quad \therefore 1 = t$ $\therefore I = \int_0^1 t\,dt$ $\therefore I = \left(\frac{t^2}{2}\right)_0^1$ $\therefore I = \frac{1}{2} - \frac{0}{2}$ $\therefore I = \frac{1}{2}$
5	$Let\ I = \int_0^1 \frac{dx}{x^2 + x + 1}$ $Third\ term = \left(\frac{1}{2} \times 1\right)^2 = \frac{1}{4}$ $\therefore I = \int_0^1 \frac{dx}{x^2 + x + \frac{1}{4} + 1 - \frac{1}{4}}$ $\therefore I = \int_0^1 \frac{dx}{\left(x + \frac{1}{2}\right)^2 + \frac{3}{4}}$
6	$Let\ I = \int_0^{\pi} cos^3x\ sinx\ dx$ $Put\ cosx = t$ $sinx\ dx = -dt$ $At\ x = 0, \quad cos0 = t, \quad 1 = t$ $At\ x = \pi, \quad , \quad cos\pi = t \quad -1 = t$ $\therefore I = \int_1^{-1} t^3\ dt$ $\therefore I = \left(\frac{t^4}{4}\right)_1^{-1}$ $\therefore I = \frac{(-1)^4}{4} - \frac{1^4}{4}$ $\therefore I = 0$
7	$Let\ I = \int_0^{\pi/2} sin^3x\ dx$ $\therefore I = \int_0^{\pi/2} \frac{3sinx - sin3x}{4} dx$ $\therefore I = \frac{1}{4}\left[3\int_0^{\pi/2} sinx\ dx - \int_0^{\pi/2} sin3x\ dx\right]$

	$\therefore I = \frac{1}{4}\left[(-cosx)_0^{\pi/2} - \left(\frac{cos3x}{3}\right)_0^{\pi/2}\right]$ $\therefore I = \frac{1}{4}\left[\left(-cos\,{}^{\pi}/_{2} + cos0\right) + \left(\frac{cos3 \times \frac{\pi}{2}}{3} - \frac{cos3 \times 0}{3}\right)\right]$ $\therefore I = \frac{1}{4}\left[1 - \frac{1}{3}\right]$ $\therefore I = \frac{1}{6}$
8	$Let\ I = \int_1^e \frac{1}{x} logx\ dx$ $Put\ logx = t$ $\therefore \frac{1}{x} dx = dt$ $At\ x = 1, \quad log1 = t, \quad 0 = t$ $At\ x = e, \quad loge = t, \quad 1 = t$ $\therefore I = \int_0^1 t\,dt$ $\therefore I = \left(\frac{t^2}{2}\right)_0^1$ $\therefore I = \frac{1}{2} - \frac{0}{2}$ $\therefore I = \frac{1}{2}$

9	Evaluate $\int_0^1 x.\sin^{-1} x dx$ $\int_0^1 x.\sin^{-1} x dx$ $= \sin^{-1} x \int_0^1 x.dx - \int_0^1 \left[\int_0^1 x.dx \frac{d(\sin^{-1} x)}{dx} \right] dx$ $= \sin^{-1} x.\frac{x^2}{2} - \int_0^1 \frac{x^2}{2}.\frac{1}{\sqrt{1-x^2}} dx$ $= \frac{x^2.\sin^{-1} x}{2} + \frac{1}{2}\int_0^1 \frac{1-x^2-1}{\sqrt{1-x^2}} dx$ $= \frac{x^2.\sin^{-1} x}{2} + \frac{1}{2}\left[\int_0^1 \left(\frac{1-x^2}{\sqrt{1-x^2}} - \frac{1}{\sqrt{1-x^2}} \right) dx \right]$ $= \frac{x^2.\sin^{-1} x}{2} + \frac{1}{2}\left[\int_0^1 \left(\sqrt{1-x^2} - \frac{1}{\sqrt{1-x^2}} \right) dx \right]$ $= \left[\frac{x^2.\sin^{-1} x}{2} + \frac{1}{2}\left[\frac{x}{2}\sqrt{1-x^2} + \frac{1^2}{2}\sin^{-1}(x) - \sin^{-1} x \right] \right]_0^1$ $= \left[\frac{x^2.\sin^{-1} x}{2} + \frac{x}{4}\sqrt{1-x^2} - \frac{1}{4}\sin^{-1}(x) \right]_0^1$ $= \left[\frac{1^2.\sin^{-1}(1)}{2} + 0 - \frac{1}{4}\sin^{-1}(1) \right] - 0$ $= \frac{\frac{\pi}{2}}{2} - \frac{1}{4}.\frac{\pi}{2} = \frac{\pi}{4} - \frac{\pi}{8} = \frac{\pi}{8}$
10	$\int_0^{\frac{\pi}{2}} \frac{\sin(2x)}{4-\sin^2 x} dx$ $= \int_0^{\frac{\pi}{2}} \frac{2\sin x \cos x}{4-\sin^2 x} dx$ Put $\sin x = t$ when $x = 0$ $t = 0$ $\cos x dx = dt$ when $x = \frac{\pi}{2}$ $t = 1$ $= \int_0^1 \frac{2t dt}{4-t^2}$ $= -\int_0^1 \frac{-2t dt}{4-t^2}$ $= -\left[\log(4-t^2) \right]_0^1$ $= -\left[\log(4-1^2) - \log(4-0^2) \right]$ $= -\log(3) + \log(4)$
11	$Let\ I = \int_0^{\pi/2} \frac{dx}{5+4cosx}$ $Put\ tan\frac{x}{2} = t, \quad dx = \frac{2dt}{1+t^2}, \quad cosx = \frac{1-t^2}{1+t^2}$ $At\ x = \pi/2, \quad tan\frac{\pi/2}{2} = t, \quad 1 = t$ $At\ x = 0, \quad tan0 = t, \quad 0 = t$

	$\therefore I = \int_0^1 \frac{\frac{2dt}{1+t^2}}{5+4\frac{1-t^2}{1+t^2}}$ $\therefore I = 2\int_0^1 \frac{1}{t^2+9}dt$ $\therefore I = 2\left(\frac{1}{3}\tan^{-1}\frac{t}{3}\right)_0^1$ $\therefore I = \frac{2}{3}\left[\tan^{-1}\frac{1}{3} - \tan^{-1}0\right]$ $\therefore I = \frac{2}{3}\tan^{-1}\frac{1}{3}$
12	$Let\ I = \int_0^{\pi/2} sin3x.cos3x\ \ dx$ $Put\ sin3x = t$ $cos3x\ dx = \frac{dt}{3}$ $At\ x = 0, \quad sin0 = t, \quad 0 = t$ $At\ x = \pi/2, \quad sin\frac{3\pi}{2} = t, \quad 1 = t$ $\therefore I = \frac{1}{3}\int_0^1 t\ dt$ $\therefore I = \frac{1}{3}\left(\frac{t^2}{2}\right)_0^1$ $\therefore I = \frac{1}{3}\left[\frac{1}{2} - 0\right]$ $\therefore I = \frac{1}{6}$
13	$\int_0^a 3x^2dx = 8$ $\therefore 3\left(\frac{x^3}{3}\right)_0^a = 8$ $a^3 = 8$ $a = 2$
14	$Let\ I = \int_0^1 \frac{dx}{1-x+x^2}$ $Third\ term = \frac{1}{4}$ $\therefore I = \int_0^1 \frac{1}{x^2 - x + \frac{1}{4} + 1 - \frac{1}{4}}dx$ $\therefore I = \int_0^1 \frac{1}{\left(x-\frac{1}{2}\right)^2 + \frac{3}{4}}dx$

	$\therefore I = \left[\frac{1}{\sqrt{3}/2}\tan^{-1}\frac{x-\frac{1}{2}}{\sqrt{3}/2}\right]_0^1$ $\therefore I = \frac{2}{\sqrt{3}}\left[\tan^{-1}\frac{1}{\sqrt{3}} - \tan^{-1}\frac{-1}{\sqrt{3}}\right]$ $\therefore I = \frac{2}{\sqrt{3}}.\frac{\pi}{3}$ $\therefore I = \frac{2\pi}{3\sqrt{3}}$
15	Evaluate: $\int_0^{\pi/2} \log(\sin x)\,dx$ Let $I = \int_0^{\pi/2} \log(\sin x)\,dx$ ----------------(1) By property $\int_0^{2a} f(x)\,dx = \int_0^{a} f(x)\,dx + \int_0^{a} f(2a-x)\,dx$ $I = \int_0^{\pi/4} \log(\sin x)\,dx + \int_0^{\pi/4} \log\left(\sin\left(\frac{\pi}{2}-x\right)\right)dx$ $I = \int_0^{\pi/4} \log(\sin x)\,dx + \int_0^{\pi/4} \log(\cos x)\,dx$

	$I=\int_0^{\pi/4}\log(\sin x.\cos x)\,dx$ $\therefore I=\int_0^{\pi/4}\log\left(\frac{2\sin x.\cos x}{2}\right)dx$ $\therefore I=\int_0^{\pi/4}\log\left(\frac{\sin 2x}{2}\right)dx$ $\therefore I=\int_0^{\pi/4}\log(\sin 2x)-\log(2)\,dx$ $\therefore I=\int_0^{\pi/4}\log(\sin 2x)\,dx-\int_0^{\pi/4}\log(2)\,dx$ Put $2x=t$ $\therefore dx=\frac{dt}{2}$ when $x\to 0$ to $\pi/4$ $t\to 0$ to $\pi/2$ $I=\int_0^{\pi/2}\log(\sin t)\frac{dt}{2}-\log 2\left[\frac{\pi}{4}-0\right]$ $I=\frac{1}{2}I-\frac{\pi\log 2}{4}$ $\therefore I-\frac{1}{2}I=-\frac{\pi\log 2}{4}$ $\frac{I}{2}=-\frac{\pi\log 2}{4}$ $\therefore I=-\frac{\pi\log 2}{2}$
16	Evaluate $\int_2^4\frac{1}{2x+3}dx$ $\int_2^4\frac{1}{2x+3}dx$ $=\frac{1}{2}[\log(2x+3)]_2^4$ $=\frac{1}{2}[\log(2(4)+3)-\log(2(2)+3)]$ $=\frac{1}{2}[\log 11-\log 7]$ or $\frac{1}{2}\log\left(\frac{11}{7}\right)$

17	Evaluate $\int_0^{\pi/2} \frac{1}{3+4\cos x}\,dx$ $\int_0^{\pi/2} \frac{1}{3+4\cos x}\,dx$ Put $\tan\frac{x}{2}=t$, $dx=\frac{2dt}{1+t^2}$, $\cos x=\frac{1-t^2}{1+t^2}$ when $x \to 0$ to $\pi/2$ $t \to 0$ to 1 $\int_0^{\pi/2} \frac{1}{3+4\cos x}\,dx = \int_0^1 \frac{\frac{2dt}{1+t^2}}{3+4\left(\frac{1-t^2}{1+t^2}\right)}$ $= \int_0^1 \frac{2dt}{3(1+t^2)+4(1-t^2)}$ $= 2\int_0^1 \frac{dt}{7-t^2} = 2\int_0^1 \frac{dt}{\left(\sqrt{7}\right)^2 - t^2}$ $= 2\left[\frac{1}{2\sqrt{7}}\log\left(\frac{\sqrt{7}+t}{\sqrt{7}-t}\right)\right]_0^1$ $= \frac{1}{\sqrt{7}}\left[\log\left(\frac{\sqrt{7}+1}{\sqrt{7}-1}\right) - \log\left(\frac{\sqrt{7}}{\sqrt{7}}\right)\right]$ $= \frac{1}{\sqrt{7}}\log\left(\frac{\sqrt{7}+1}{\sqrt{7}-1}\right)$
18	Evaluate: $\int_0^{\frac{\pi}{2}} \sin 5x \cos 3x\, dx$ $\int_0^{\frac{\pi}{2}} \sin 5x \cos 3x\, dx$ $= \frac{1}{2}\int_0^{\frac{\pi}{2}} 2\sin 5x \cos 3x\, dx$ $= \frac{1}{2}\int_0^{\frac{\pi}{2}} \left(\sin(5x+3x)+\sin(5x-3x)\right)\, dx$ $= \frac{1}{2}\int_0^{\frac{\pi}{2}} (\sin 8x + \sin 2x)\, dx$ $= \frac{1}{2}\left[\frac{-\cos 8x}{8} - \frac{\cos 2x}{2}\right]_0^{\frac{\pi}{2}}$

	$=-\frac{1}{2}\left[\frac{\cos 8x}{8}+\frac{\cos 2x}{2}\right]_0^{\frac{\pi}{2}}$ $=-\frac{1}{2}\left[\frac{\cos 8\left(\frac{\pi}{2}\right)}{8}+\frac{\cos 2\left(\frac{\pi}{2}\right)}{2}-\frac{\cos 0}{8}-\frac{\cos 0}{2}\right]$ $=-\frac{1}{2}\left[\frac{\cos 4\pi}{8}+\frac{\cos \pi}{2}-\frac{\cos 0}{8}-\frac{\cos 0}{2}\right]$ $=-\frac{1}{2}\left[\frac{1}{8}+\frac{(-1)}{2}-\frac{1}{8}-\frac{1}{2}\right]=\frac{1}{2}$
19	Evaluate $\int_1^2 \frac{dx}{4x-1}$ $\int_1^2 \frac{dx}{4x-1}$ $=\frac{1}{4}\left[\log(4x-1)\right]_1^2$ $=\frac{1}{4}\left[\log(4(2)-1)-\log(4(1)-1)\right]$ $=\frac{1}{4}[\log 7-\log 3]$ or $\frac{1}{4}\log\left(\frac{7}{3}\right)$
20	Evaluate $\int_0^1 x\cdot\tan^{-1}x dx$ $\int_0^1 x\cdot\tan^{-1}x dx$ $=\left[\tan^{-1}x\int x dx-\int\left(\int x dx\right)\frac{d}{dx}\left(\tan^{-1}x\right)dx\right]_0^1$ $=\left[\frac{x^2}{2}\tan^{-1}x-\int\frac{x^2}{2}\cdot\frac{1}{x^2+1}\cdot dx\right]_0^1$ $=\left[\frac{x^2}{2}\tan^{-1}x-\frac{1}{2}\int\frac{x^2}{x^2+1}\cdot dx\right]_0^1$ $=\left[\frac{x^2}{2}\tan^{-1}x-\frac{1}{2}\int\frac{(x^2+1)-1}{x^2+1}\cdot dx\right]_0^1$ $=\left[\frac{x^2}{2}\tan^{-1}x-\frac{1}{2}\int\left(1-\frac{1}{x^2+1}\right)\cdot dx\right]_0^1$ $=\left[\frac{x^2}{2}\tan^{-1}x-\frac{1}{2}\left(x-\tan^{-1}x\right)\right]_0^1$ $=\left[\frac{1^2}{2}\tan^{-1}1-\frac{1}{2}\left(1-\tan^{-1}1\right)\right]-[0]$ $=\frac{\pi}{8}-\frac{1}{2}\left(1-\frac{\pi}{4}\right)=\frac{\pi}{4}-\frac{1}{2}$ or 0.2854

21	Evaluate $\int_1^2 \frac{dx}{2x+5}$ $\int_1^2 \frac{dx}{2x+5}$ $=\frac{1}{2}\left[\log(2x+5)\right]_1^2$ $=\frac{1}{2}\left[\log(2(2)+5)-\log(2(1)+5)\right]$ $=\frac{1}{2}[\log 9-\log 7]$ or $\frac{1}{2}\log\left(\frac{9}{7}\right)$
22	Evaluate : $\int_0^{\pi} \frac{x\sin x}{1+\cos^2 x}\,dx$ Let $I=\int_0^{\pi} \frac{x\sin x}{1+\cos^2 x}\,dx$ ------------------(1) $\therefore I=\int_0^{\pi} \frac{(\pi-x)\sin(\pi-x)}{1+\cos^2(\pi-x)}\,dx$ $\therefore I=\int_0^{\pi} \frac{(\pi-x)\sin x}{1+\cos^2 x}\,dx$ $\therefore I=\int_0^{\pi} \frac{\pi\sin x}{1+\cos^2 x}\,dx-\int_0^{\pi} \frac{x\sin x}{1+\cos^2 x}\,dx$ $\therefore I=\int_0^{\pi} \frac{\pi\sin x}{1+\cos^2 x}\,dx-I$ $\therefore 2I=\pi\int_0^{\pi} \frac{\sin x}{1+\cos^2 x}\,dx$ Put $\cos x=t$ $\therefore -\sin x dx=dt$ $\therefore \sin x dx=-dt$ $x\to 0$, $t\to 1$ $x\to\pi$, $t\to -1$ $\therefore 2I=\pi\int_1^{-1} \frac{1}{1+t^2}\,(-dt)$ $\therefore 2I=-\pi\int_1^{-1} \frac{1}{1+t^2}\,dt$ $\therefore 2I=-\pi\left(\tan^{-1}t\right)_1^{-1}$ $\therefore 2I=-\pi\left(\tan^{-1}(-1)-\tan^{-1}1\right)$ $\therefore 2I=-\pi\left(-\frac{\pi}{4}-\frac{\pi}{4}\right)$ $\therefore I=\frac{\pi^2}{4}$

23	Evaluate $\int_0^{\frac{\pi}{4}} \log(1+\tan x)\,dx$ Let $I = \int_0^{\frac{\pi}{4}} \log(1+\tan x)\,dx$ $I = \int_0^{\frac{\pi}{4}} \log\left(1+\tan\left(\frac{\pi}{4}-x\right)\right)dx$ $I = \int_0^{\frac{\pi}{4}} \log\left(1+\frac{\tan\frac{\pi}{4}-\tan x}{1+\tan\frac{\pi}{4}\tan x}\right)dx$ $I = \int_0^{\frac{\pi}{4}} \log\left(1+\frac{1-\tan x}{1+\tan x}\right)dx$ $I = \int_0^{\frac{\pi}{4}} \log\left(\frac{1+\tan x+1-\tan x}{1+\tan x}\right)dx$ $I = \int_0^{\frac{\pi}{4}} \log\left(\frac{2}{1+\tan x}\right)dx$ $I = \int_0^{\frac{\pi}{4}} [\log 2-\log(1+\tan x)]dx$ $I = \log 2\int_0^{\frac{\pi}{4}} dx - \int_0^{\frac{\pi}{4}} \log(1+\tan x)\,dx$ $I = \log 2[x]_0^{\frac{\pi}{4}} - I$ $2I = \log 2\left[\frac{\pi}{4}-0\right]$ $I = \frac{\pi}{8}\log 2$
24	$Let\ I = \int_0^1 \frac{1-x}{1+x}dx$ $\therefore I = \int_0^1 \frac{-(x-1+1-1)}{x+1}dx$ $\therefore I = \int_0^1 \frac{-[(x+1)-2]}{x+1}dx$ $\therefore I = \int_0^1 \left[\frac{-(x+1)+2}{x+1}\right]dx$ $\therefore I = \int_0^1 \left[-1+\frac{2}{x+1}\right]dx$ $\therefore I = -\int_0^1 1\,dx + 2\int_0^1 \frac{1}{x+1}dx$ $\therefore I = [-x]_0^1 + 2[log(x+1)]_0^1$ $\therefore I = -1+2log2$

25	Let $I=\int_0^1 \frac{e^x-e^{-x}}{e^x+e^{-x}}dx$ Put $e^x+e^{-x}=t$ $\therefore (e^x-e^{-x})dx=dt$ As $x\to 0\ \&\ 1,\quad t\to 2\ \&\ e+\frac{1}{e}$ $\therefore I=\int_2^{e+\frac{1}{e}} \frac{1}{t}dt$ $\therefore I=(logt)_2^{e+\frac{1}{e}}$ $\therefore I=log\frac{e^2+1}{e}-log2$
26	Let $I=\int_0^{\pi/2} \frac{cosx}{1+sin^2x}dx$ Put $sinx=t$ $\therefore cosxdx=dt$ As $x\to 0\ \&\ \frac{\pi}{2},\quad t\to 0\ \&1$ $\therefore I=\int_0^1 \frac{1}{1+t^2}dt$ $\therefore I=[\tan^{-1}t]_0^1$ $\therefore I=\frac{\pi}{4}$
27	Evaluate $\int_0^{\pi}\frac{dx}{5+4\cos x}$ Put $\tan\frac{x}{2}=t$ $\therefore dx=\frac{2dt}{1+t^2}$ and $\cos x=\frac{1-t^2}{1+t^2}$ <table><tr><td>x</td><td>0</td><td>π</td></tr><tr><td>t</td><td>0</td><td>∞</td></tr></table> $\therefore \int_0^{\pi}\frac{dx}{5+4\cos x}=\int_0^{\infty}\frac{1}{5+4\left(\frac{1-t^2}{1+t^2}\right)}\cdot\frac{2dt}{1+t^2}$ $=2\int_0^{\infty}\frac{1}{t^2+9}dt$ $=2\int_0^{\infty}\frac{1}{t^2+3^2}dt$ $=2\times\frac{1}{3}\left[\tan^{-1}\left(\frac{t}{3}\right)\right]_0^{\infty}$ $=\frac{2}{3}\left[\tan^{-1}\infty-\tan^{-1}0\right]$ $=\frac{2}{3}\left[\frac{\pi}{2}\right]$ $=\frac{\pi}{3}$

28	$Let\ I = \int_0^{\pi/2} \frac{dx}{\sqrt{9-4x^2}}$ $\therefore I = \int_0^{\pi/2} \frac{1}{\sqrt{4(\frac{9}{4}-x^2)}} dx$ $\therefore I = \frac{1}{2}\int_0^{\pi/2} \frac{1}{\sqrt{\left(\frac{3}{2}\right)^2 - x^2}} dx$ $\therefore I = \frac{1}{2}\left[\sin^{-1}\left(\frac{x}{3/2}\right)\right]_0^{\pi/2}$ $\therefore I = \frac{1}{2}\left[\frac{1}{2} - 0\right]$ $\therefore I = \frac{1}{4}$
29	$Let\ I = \int_{-1}^{1} \frac{sinx}{1+cosx} dx$ $Put\ 1 + cosx = t$ $\therefore sinxdx = -dt$ $\therefore I = 2\int_1^1 \frac{1}{t} dt$ $\therefore I = 2(logt)_1^1$ $\therefore I = 2log1 - 2log(1)$ $\therefore I = 0$
30	Evaluate $\int_0^{\frac{\pi}{2}} \frac{dx}{4+5\cos x}$ Put $\tan\frac{x}{2} = t$ $\cos x = \frac{1-t^2}{1+t^2}$, $dx = \frac{2dt}{1+t^2}$ when $x \to 0$ to $\frac{\pi}{2}$ $t \to 0$ to 1 $\therefore I = \int_0^1 \frac{1}{4+5\left(\frac{1-t^2}{1+t^2}\right)} \frac{2dt}{1+t^2}$ $I = 2\int_0^1 \frac{1}{4(1+t^2)+5(1-t^2)} dt$ $I = 2\int_0^1 \frac{1}{4+4t^2+5-5t^2} dt$

	$I=2\int_0^1 \frac{1}{9-t^2}dt$ $I=2\int_0^1 \frac{1}{(3)^2-t^2}dt$ $I=2\left[\frac{1}{2(3)}\log\left\|\frac{3+t}{3-t}\right\|\right]_0^1$ $I=\frac{1}{3}\left[\log\left\|\frac{4}{2}\right\|-\log\left\|\frac{3}{3}\right\|\right]$ $I=\frac{1}{3}\left[\log\|2\|-\log\|1\|\right]$ $I=\frac{1}{3}\log\|2\|$
31	Evaluate $\int_1^5 \frac{\sqrt[3]{9-x}}{\sqrt[3]{9-x}+\sqrt[3]{x+3}}dx$ Let $I=\int_1^5 \frac{\sqrt[3]{9-x}}{\sqrt[3]{9-x}+\sqrt[3]{x+3}}dx$ ....(1) Using property $I=\int_1^5 \frac{\sqrt[3]{9-(1+5-x)}}{\sqrt[3]{9-(1+5-x)}+\sqrt[3]{1+5-x+3}}dx$ $I=\int_1^5 \frac{\sqrt[3]{x+3}}{\sqrt[3]{x+3}+\sqrt[3]{9-x}}dx$ ....(2) adding equation (1) and (2) $\therefore 2I=\int_1^5 \frac{\sqrt[3]{9-x}+\sqrt[3]{x+3}}{\sqrt[3]{x+3}+\sqrt[3]{9-x}}dx$ $\therefore 2I=\int_1^5 1dx$ $=[x]_1^5$ $=5-1=4$ $\therefore I=2$
32	Evaluate $\int_0^{\frac{\pi}{2}} \frac{1}{1+\sqrt{\cot x}}dx$ Let $I=\int_0^{\frac{\pi}{2}} \frac{1}{1+\sqrt{\cot x}}dx$ $=\int_0^{\frac{\pi}{2}} \frac{1}{1+\sqrt{\frac{\cos x}{\sin x}}}dx$ $\therefore I=\int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx$ ----------------(1)

	$I=\int_0^{\frac{\pi}{2}}\frac{\sqrt{\sin\left(\frac{\pi}{2}-x\right)}}{\sqrt{\sin\left(\frac{\pi}{2}-x\right)}+\sqrt{\cos\left(\frac{\pi}{2}-x\right)}}dx$ $\therefore I=\int_0^{\frac{\pi}{2}}\frac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}}dx$ ----------------(2) Add (1) and (2) $I+I=\int_0^{\frac{\pi}{2}}\frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx+\int_0^{\frac{\pi}{2}}\frac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}}dx$ $2I=\int_0^{\frac{\pi}{2}}\frac{\sqrt{\sin x}+\sqrt{\cos x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx$ $2I=\int_0^{\frac{\pi}{2}}1dx$ $2I=[x]_0^{\frac{\pi}{2}}$ $2I=\frac{\pi}{2}-0$ $I=\frac{\pi}{4}$
33	$I=\int_3^7\frac{(10-x)^2}{x^2+(10-x)^2}\ dx$ --------(1) $I=\int_3^7\frac{[10-(10-x)]^2}{(10-x)^2+[10-(10-x)]^2}\ dx$ $I=\int_3^7\frac{x^2}{(10-x)^2+x^2}\ dx$ --------(2) Adding (1) and (2) $2I=\int_3^7\frac{(10-x)^2+x^2}{(10-x)^2+x^2}dx$ $2I=\int_3^7 1\,dx$ $2I=[x]_3^7=7-3=4$ $I=2$

34	
	Evaluate: $\int_0^{\frac{\pi}{2}} \frac{\tan x}{1+\tan x}dx$ Let $I = \int_0^{\frac{\pi}{2}} \frac{\tan x}{1+\tan x}dx$ $= \int_0^{\frac{\pi}{2}} \frac{\frac{\sin x}{\cos x}}{1+\frac{\sin x}{\cos x}}dx$ $\therefore I = \int_0^{\pi/2} \frac{\sin x}{\sin x+\cos x}\,dx$ ----------------(1) $I = \int_0^{\pi/2} \frac{\sin\left(\pi/2 - x\right)}{\sin\left(\pi/2 - x\right)+\cos\left(\pi/2 - x\right)}\,dx$ $\therefore I = \int_0^{\pi/2} \frac{\cos x}{\cos x+\sin x}\,dx$ ----------------(2) add (1) and (2) $I + I = \int_0^{\pi/2} \frac{\sin x}{\sin x+\cos x}\,dx + \int_0^{\pi/2} \frac{\cos x}{\cos x+\sin x}\,dx$ $2I = \int_0^{\pi/2} \frac{\sin x+\cos x}{\sin x+\cos x}\,dx$ $2I = \int_0^{\pi/2} 1\,dx$ $2I = [x]_0^{\pi/2}$ $2I = \frac{\pi}{2} - 0$ $I = \frac{\pi}{4}$

35	Evaluate $\int_0^7 \frac{\sqrt[3]{x}}{\sqrt[3]{x}+\sqrt[3]{7-x}}\,dx$ Let $I=\int_0^7 \frac{\sqrt[3]{x}}{\sqrt[3]{x}+\sqrt[3]{7-x}}\,dx$ ------------------(1) $I=\int_0^7 \frac{\sqrt[3]{7-x}}{\sqrt[3]{7-x}+\sqrt[3]{7-(7-x)}}\,dx$ $\therefore I=\int_0^7 \frac{\sqrt[3]{7-x}}{\sqrt[3]{7-x}+\sqrt[3]{x}}\,dx$------------------(2) add (1) and (2) $\therefore I+I=\int_0^7 \frac{\sqrt[3]{x}}{\sqrt[3]{x}+\sqrt[3]{7-x}}\,dx+\int_0^7 \frac{\sqrt[3]{7-x}}{\sqrt[3]{7-x}+\sqrt[3]{x}}\,dx$ $\therefore 2I=\int_0^7 \frac{\sqrt[3]{x}+\sqrt[3]{7-x}}{\sqrt[3]{x}+\sqrt[3]{7-x}}\,dx$ $\therefore 2I=\int_0^7 1\,dx$ $\therefore 2I=[x]_0^7$ $\therefore 2I=7-0$ $\therefore I=\frac{7}{2}$
36	$Let\ I=\int_0^{\pi/2} \frac{\sqrt[3]{secx}}{\sqrt[3]{secx}+\sqrt[3]{cosecx}}\,dx$-------------(1) $Replace\ x\ by\ a+b--x=0+\frac{\pi}{2}-x=\frac{\pi}{2}-x$ $\therefore I=\int_0^{\pi/2} \frac{\sqrt[3]{\sec(\frac{\pi}{2}-x)}}{\sqrt[3]{\sec(\frac{\pi}{2}-x)}+\sqrt[3]{cosec(\frac{\pi}{2}-x)}}\,dx$ $\therefore I=\int_0^{\pi/2} \frac{\sqrt[3]{cosecx}}{\sqrt[3]{cosecx}+\sqrt[3]{secx}}\,dx$---------------(2) $add\ (1)\ \&\ (2),$ $\therefore I+I=\int_0^{\pi/2} \frac{\sqrt[3]{secx}}{\sqrt[3]{secx}+\sqrt[3]{cosecx}}\,dx+\int_0^{\pi/2} \frac{\sqrt[3]{cosecx}}{\sqrt[3]{cosecx}+\sqrt[3]{secx}}\,dx$ $\therefore 2I=\int_0^{\pi/2} \frac{\sqrt[3]{secx}+\sqrt[3]{cosecx}}{\sqrt[3]{secx}+\sqrt[3]{cosecx}}\,dx$ $\therefore 2I=\int_0^{\pi/2} 1\,dx$ $\therefore 2I=[x]_0^{\pi/2}$ $\therefore 2I=\frac{\pi}{2}$ $\therefore I=\frac{\pi}{4}$

37	Evaluate: $\int_1^4 \frac{\sqrt[3]{9-x}}{\sqrt[3]{9-x}+\sqrt[3]{x+4}}\,dx$ $I=\int_1^4 \frac{\sqrt[3]{9-x}}{\sqrt[3]{9-x}+\sqrt[3]{x+4}}\,dx$ -----------------(1) $I=\int_1^4 \frac{\sqrt[3]{9-(5-x)}}{\sqrt[3]{9-(5-x)}+\sqrt[3]{(5-x)+4}}\,dx$ $\therefore I=\int_1^4 \frac{\sqrt[3]{x+4}}{\sqrt[3]{x+4}+\sqrt[3]{9-x}}\,dx$-----------------(2) add (1) and (2), $I+I=\int_1^4 \frac{\sqrt[3]{9-x}}{\sqrt[3]{9-x}+\sqrt[3]{x+4}}\,dx+\int_1^4 \frac{\sqrt[3]{x+4}}{\sqrt[3]{x+4}+\sqrt[3]{9-x}}\,dx$ $\therefore 2I=\int_1^4 \frac{\sqrt[3]{9-x}+\sqrt[3]{x+4}}{\sqrt[3]{9-x}+\sqrt[3]{x+4}}\,dx$ $\therefore 2I=\int_1^4 1\,dx$ $\therefore 2I=(x)_1^4$ $\therefore I=\frac{3}{2}$
38	Evaluate : $\int_0^{\pi/2} \frac{\sqrt[3]{\sin x}}{\sqrt[3]{\cos x}+\sqrt[3]{\sin x}}dx$ $I=\int_0^{\pi/2} \frac{\sqrt[3]{\sin x}}{\sqrt[3]{\cos x}+\sqrt[3]{\sin x}}dx$---------(1) $=\int_0^{\pi/2} \frac{\sqrt[3]{\sin\left(\frac{\pi}{2}-x\right)}}{\sqrt[3]{\cos\left(\frac{\pi}{2}-x\right)}+\sqrt[3]{\sin\left(\frac{\pi}{2}-x\right)}}dx$ $I=\int_0^{\pi/2} \frac{\sqrt[3]{\cos x}}{\sqrt[3]{\sin x}+\sqrt[3]{\cos x}}dx$---------(2) Add (1) and (2) $\therefore 2I=\int_0^{\pi/2} \frac{\sqrt[3]{\sin x}+\sqrt[3]{\cos x}}{\sqrt[3]{\sin x}+\sqrt[3]{\cos x}}dx$ $=\int_0^{\pi/2} 1\cdot dx$ $=[x]_0^{\pi/2}$ $2I=\frac{\pi}{2}$ $\therefore I=\frac{\pi}{4}$ *OR* $I=\int_0^{\pi/2} \frac{\sqrt[3]{\sin x}}{\sqrt[3]{\cos x}+\sqrt[3]{\sin x}}dx$ Replace $x\rightarrow\frac{\pi}{2}-x$, $\therefore \sin x\rightarrow\cos x$ & $\cos x\rightarrow\sin x$ $\therefore I=\int_0^{\pi/2} \frac{\sqrt[3]{\cos x}}{\sqrt[3]{\sin x}+\sqrt[3]{\cos x}}dx$ $\therefore 2I=\int_0^{\pi/2} \frac{\sqrt[3]{\sin x}+\sqrt[3]{\cos x}}{\sqrt[3]{\sin x}+\sqrt[3]{\cos x}}dx$ $=\int_0^{\pi/2} 1\cdot dx$

	$=[x]_0^{\pi/2}$ $2I=\dfrac{\pi}{2}$ $\therefore I=\dfrac{\pi}{4}$
39	Let $I=\int_0^5 \dfrac{\sqrt{9-x}}{\sqrt{9-x}+\sqrt{x+4}}\,dx$ ------------------(1) $I=\int_0^5 \dfrac{\sqrt{9-(5-x)}}{\sqrt{9-(5-x)}+\sqrt{5-x+4}}\,dx$ $\therefore I=\int_0^5 \dfrac{\sqrt{9-5+x}}{\sqrt{\sqrt{9-5+x}+\sqrt{9-x}}}\,dx$ $\therefore I=\int_0^5 \dfrac{\sqrt{4+x}}{\sqrt{\sqrt{4+x}+\sqrt{9-x}}}\,dx$ ------------------(2) add (1) and (2) $\therefore I+I=\int_0^5 \dfrac{\sqrt{9-x}}{\sqrt{9-x}+\sqrt{x+4}}\,dx+\int_0^5 \dfrac{\sqrt{4+x}}{\sqrt{\sqrt{4+x}+\sqrt{9-x}}}\,dx$ $\therefore 2I=\int_0^5 \dfrac{\sqrt{9-x}+\sqrt{4+x}}{\sqrt{9-x}+\sqrt{x+4}}\,dx$ $\therefore 2I=\int_0^5 1\,dx$ $\therefore 2I=[x]_0^5$ $\therefore 2I=5-0$ $\therefore I=\dfrac{5}{2}$
40	Evaluate: $\int_1^3 \dfrac{\sqrt[3]{x+5}}{\sqrt[3]{x+5}+\sqrt[3]{9-x}}\,dx$ $I=\int_1^3 \dfrac{\sqrt[3]{x+5}}{\sqrt[3]{x+5}+\sqrt[3]{9-x}}\,dx$ ------------------(1) $I=\int_1^3 \dfrac{\sqrt[3]{(1+3-x)+5}}{\sqrt[3]{(1+3-x)+5}+\sqrt[3]{9-(1+3-x)}}\,dx$ $\therefore I=\int_1^3 \dfrac{\sqrt[3]{9-x}}{\sqrt[3]{9-x}+\sqrt[3]{x+5}}\,dx$ ------------------(2) add (1) and (2) $I+I=\int_1^3 \dfrac{\sqrt[3]{x+5}}{\sqrt[3]{x+5}+\sqrt[3]{9-x}}\,dx+\int_1^3 \dfrac{\sqrt[3]{9-x}}{\sqrt[3]{9-x}+\sqrt[3]{x+5}}\,dx$ $2I=\int_1^3 \dfrac{\sqrt[3]{x+5}+\sqrt[3]{9-x}}{\sqrt[3]{9-x}+\sqrt[3]{x+5}}\,dx$ $2I=\int_1^3 1\,dx$ $2I=[x]_1^3$ $2I=3-1$ $I=1$

41	$Let\ I = \int_1^3 \frac{\sqrt[3]{x+5}}{\sqrt[3]{x+5}+\sqrt[3]{9-x}}\ dx$---------(1) $Replace\ x\ by\ 4-x,$ $\therefore I = \int_1^3 \frac{\sqrt[3]{9-x}}{\sqrt[3]{9-x}+\sqrt[3]{x+5}}\ dx$-----------(2) $add\ (1)\ \&\ (2),$ $\therefore I + I = \int_1^3 \frac{\sqrt[3]{x+5}}{\sqrt[3]{x+5}+\sqrt[3]{9-x}}\ dx + \int_1^3 \frac{\sqrt[3]{9-x}}{\sqrt[3]{9-x}+\sqrt[3]{x+5}}\ dx$ $\therefore 2I = \int_1^3 \frac{\sqrt[3]{x+5}+\sqrt[3]{9-x}}{\sqrt[3]{x+5}+\sqrt[3]{9-x}}\ dx$ $\therefore 2I = \int_1^3 1\,dx$ $\therefore 2I = [x]_1^3$ $\therefore 2I = 3-1$ $\therefore I = 1$
42	Evaluate, $\int_0^{\pi/2} \frac{dx}{1+\tan x}$ $\int_0^{\pi/2} \frac{dx}{1+\tan x}$ $\therefore I = \int_0^{\pi/2} \frac{1}{1+\frac{\sin x}{\cos x}}\ dx$ $\therefore I = \int_0^{\pi/2} \frac{\cos x}{\cos x+\sin x}\ dx$ ------------------(1) $\therefore I = \int_0^{\pi/2} \frac{\cos(\pi/2 - x)}{\cos(\pi/2 - x)+\sin(\pi/2 - x)}\ dx$ $\therefore I = \int_0^{\pi/2} \frac{\sin x}{\sin x+\cos x}\ dx$ ------------------(2) add (1) and (2) $\therefore I + I = \int_0^{\pi/2} \frac{\cos x}{\cos x+\sin x}\ dx + \int_0^{\pi/2} \frac{\sin x}{\sin x+\cos x}\ dx$ $2I = \int_0^{\pi/2} \frac{\cos x+\sin x}{\cos x+\sin x}\ dx$ $2I = \int_0^{\pi/2} 1\ dx$ $2I = [x]_0^{\pi/2}$ $2I = \frac{\pi}{2} - 0$ $\therefore I = \frac{\pi}{4}$

43	Evaluate $\int_0^{\pi/2} \frac{dx}{1+\cot x}$ $\int_0^{\pi/2} \frac{dx}{1+\cot x}$ $\therefore I = \int_0^{\pi/2} \frac{1}{1+\frac{\cos x}{\sin x}} dx$ $\therefore I = \int_0^{\pi/2} \frac{\sin x}{\sin x + \cos x} dx$ ------------------(1) $\therefore I = \int_0^{\pi/2} \frac{\sin\left(\pi/2 - x\right)}{\sin\left(\pi/2 - x\right) + \cos\left(\pi/2 - x\right)} dx$ $\therefore I = \int_0^{\pi/2} \frac{\cos x}{\cos x + \sin x} dx$ ------------------(2) add (1) and (2) $\therefore I + I = \int_0^{\pi/2} \frac{\sin x}{\sin x + \cos x} dx + \int_0^{\pi/2} \frac{\cos x}{\cos x + \sin x} dx$ $2I = \int_0^{\pi/2} \frac{\sin x + \cos x}{\sin x + \cos x} dx$ $2I = \int_0^{\pi/2} 1\, dx$ $2I = [x]_0^{\pi/2}$ $2I = \frac{\pi}{2} - 0$ $\therefore I = \frac{\pi}{4}$
44	$Let\ I = \int_0^{\pi/2} \frac{tanx}{tanx+cotx} dx$--------------(1) $Replace\ x\ by\ \frac{\pi}{2} - x,$ $\therefore I = \int_0^{\pi/2} \frac{cotx}{cotx+tanx} dx$--------------(2) $Add\ (1)\ \&\ (2),$ $\therefore I + I = \int_0^{\pi/2} \frac{tanx}{tanx + cotx} dx + \int_0^{\pi/2} \frac{cotx}{cotx + tanx} dx$ $\therefore 2I = \int_0^{\pi/2} \frac{tanx + cotx}{tanx + cotx} dx$ $\therefore 2I = \int_0^{\pi/2} 1\, dx$ $\therefore 2I = [x]_0^{\pi/2}$ $\therefore 2I = \frac{\pi}{2}$ $\therefore I = \frac{\pi}{4}$

45	Evaluate $\int_2^5 \frac{\sqrt{x}}{\sqrt{7-x}+\sqrt{x}}\,dx$ $I=\int_2^5 \frac{\sqrt{x}}{\sqrt{7-x}+\sqrt{x}}\,dx$ ------------------(1) $\therefore I=\int_2^5 \frac{\sqrt{(2+5-x)}}{\sqrt{7-(2+5-x)}+\sqrt{(2+5-x)}}\,dx$ $\therefore I=\int_2^5 \frac{\sqrt{7-x}}{\sqrt{x}+\sqrt{7-x}}\,dx$------------------(2) add (1) and (2) $I+I=\int_2^5 \frac{\sqrt{x}}{\sqrt{7-x}+\sqrt{x}}\,dx+\int_2^5 \frac{\sqrt{7-x}}{\sqrt{x}+\sqrt{7-x}}\,dx$ $\therefore 2I=\int_2^5 \frac{\sqrt{x}+\sqrt{7-x}}{\sqrt{7-x}+\sqrt{x}}\,dx$ $\therefore 2I=\int_2^5 1\,dx$ $\therefore 2I=[x]_2^5$ $\therefore 2I=5-2$ $\therefore 2I=3$ $\therefore I=\frac{3}{2}$
46	Evaluate $\int_0^5 \frac{\sqrt{5-x}}{\sqrt{x}+\sqrt{5-x}}\,dx$ $I=\int_0^5 \frac{\sqrt{5-x}}{\sqrt{x}+\sqrt{5-x}}\,dx$ ------------------(1) $I=\int_0^5 \frac{\sqrt{5-(5-x)}}{\sqrt{5-x}+\sqrt{5-(5-x)}}\,dx$ $\therefore I=\int_0^5 \frac{\sqrt{x}}{\sqrt{5-x}+\sqrt{x}}\,dx$ ------------------(2) add (1) and (2) $I+I=\int_0^5 \frac{\sqrt{5-x}}{\sqrt{x}+\sqrt{5-x}}\,dx+\int_0^5 \frac{\sqrt{x}}{\sqrt{5-x}+\sqrt{x}}\,dx$ $2I=\int_0^5 \frac{\sqrt{5-x}+\sqrt{x}}{\sqrt{x}+\sqrt{5-x}}\,dx$ $2I=\int_0^5 1\,dx$ $2I=[x]_0^5$ $2I=5-0$ $I=\frac{5}{2}$

47	$Let\ I = \int_{\pi/6}^{\pi/3} \frac{1}{1+\sqrt[n]{tanx}}\,dx$ $\therefore I = \int_{\pi/6}^{\pi/3} \frac{1}{1+\sqrt[n]{\frac{sinx}{cosx}}}\,dx$ $\therefore I = \int_{\pi/6}^{\pi/3} \frac{\sqrt[n]{cosx}}{\sqrt[n]{cosx}+\sqrt[n]{sinx}}\,dx$------------(1) $Replace\ x\ by\ \frac{\pi}{6}+\frac{\pi}{3}-x = \frac{\pi}{2}-x,$ $\therefore I = \int_{\pi/6}^{\pi/3} \frac{\sqrt[n]{sinx}}{\sqrt[n]{sinx}+\sqrt[n]{cosx}}\,dx$-------------(2) Add (1) & (2), $\therefore I + I = \int_{\pi/6}^{\pi/3} \frac{\sqrt[n]{cosx}}{\sqrt[n]{cosx}+\sqrt[n]{sinx}}\,dx + \int_{\pi/6}^{\pi/3} \frac{\sqrt[n]{sinx}}{\sqrt[n]{sinx}+\sqrt[n]{cosx}}\,dx$ $\therefore 2I = \int_{\pi/6}^{\pi/3} \frac{\sqrt[n]{cosx}+\sqrt[n]{sinx}}{\sqrt[n]{cosx}+\sqrt[n]{sinx}}\,dx$ $\therefore 2I = \int_{\pi/6}^{\pi/3} 1\,dx$ $\therefore 2I = [x]_{\pi/6}^{\pi/3}$ $\therefore 2I = \frac{\pi}{3} - \frac{\pi}{6}$ $\therefore I = \frac{\pi}{12}$
48	$Let\ I = \int_{0}^{\pi/2} \frac{1}{1+\sqrt{tanx}}\,dx$ $\therefore I = \int_{0}^{\pi/2} \frac{1}{1+\sqrt{\frac{sinx}{cosx}}}\,dx$ $\therefore I = \int_{0}^{\pi/2} \frac{\sqrt{cosx}}{\sqrt{cosx}+\sqrt{sinx}}\,dx$ −−−−−−−−−−−−(1) $Replace\ x\ by\ \frac{\pi}{2}-x,$ $\therefore I = \int_{0}^{\pi/2} \frac{\sqrt{sinx}}{\sqrt{sinx}+\sqrt{cosx}}\,dx$ −−−−−−−−−−−−(2) Add (1) & (2), $\therefore I + I = \int_{0}^{\pi/2} \frac{\sqrt{cosx}}{\sqrt{cosx}+\sqrt{sinx}}\,dx + \int_{0}^{\pi/2} \frac{\sqrt{sinx}}{\sqrt{sinx}+\sqrt{cosx}}\,dx$ $\therefore 2I = \int_{0}^{\pi/2} \frac{\sqrt{cosx}+\sqrt{sinx}}{\sqrt{cosx}+\sqrt{sinx}}\,dx$

$$\therefore 2I = \int_0^{\pi/2} 1\, dx$$

$$\therefore 2I = [x]_0^{\pi/2}$$

$$\therefore 2I = \frac{\pi}{2} - 0$$

$$\therefore I = \frac{\pi}{4}$$

49

Evaluate : $\int_0^{\frac{\pi}{2}} \frac{1}{1+\sqrt{\tan x}}dx$

$$I = \int_0^{\frac{\pi}{2}} \frac{1}{1+\sqrt{\tan x}}dx$$

$$= \int_0^{\frac{\pi}{2}} \frac{1}{1+\frac{\sqrt{\sin x}}{\sqrt{\cos x}}}dx$$

$$I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}}dx --------(1)$$

by property

$$\therefore I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\cos\left(\frac{\pi}{2}-x\right)}}{\sqrt{\cos\left(\frac{\pi}{2}-x\right)}+\sqrt{\sin\left(\frac{\pi}{2}-x\right)}}dx$$

$$= \int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx --------(2)$$

add (1) and (2)

$$2I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}+\sqrt{\cos x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx$$

$$2I = \int_0^{\frac{\pi}{2}} 1dx$$

$$\therefore 2I = [x]_0^{\frac{\pi}{2}}$$

$$= \frac{\pi}{2} - 0$$

$$I = \frac{\pi}{4}$$

50	Evaluate $\int_0^{\pi/2} \frac{dx}{1+\sqrt[3]{\tan x}}$

$$\int_0^{\pi/2} \frac{dx}{1+\sqrt[3]{\tan x}}$$

$$= \int_0^{\pi/2} \frac{dx}{1+\sqrt[3]{\frac{\sin x}{\cos x}}}$$

$$= \int_0^{\pi/2} \frac{dx}{1+\frac{\sqrt[3]{\sin x}}{\sqrt[3]{\cos x}}}$$

$$= \int_0^{\pi/2} \frac{dx}{\frac{\sqrt[3]{\cos x}+\sqrt[3]{\sin x}}{\sqrt[3]{\cos x}}}$$

$$I = \int_0^{\pi/2} \frac{\sqrt[3]{\cos x}}{\sqrt[3]{\cos x}+\sqrt[3]{\sin x}}dx \quad \text{--------------(1)}$$

$$= \int_0^{\pi/2} \frac{\sqrt[3]{\cos\left(\frac{\pi}{2}-x\right)}}{\sqrt[3]{\cos\left(\frac{\pi}{2}-x\right)}+\sqrt[3]{\sin\left(\frac{\pi}{2}-x\right)}}dx \quad \text{------------By property}$$

$$I = \int_0^{\pi/2} \frac{\sqrt[3]{\sin x}}{\sqrt[3]{\sin x}+\sqrt[3]{\cos x}}dx \quad \text{--------------(2)}$$

Add (1) and (2)

$$I+I = \int_0^{\pi/2} \frac{\sqrt[3]{\cos x}}{\sqrt[3]{\cos x}+\sqrt[3]{\sin x}}dx + \int_0^{\pi/2} \frac{\sqrt[3]{\sin x}}{\sqrt[3]{\sin x}+\sqrt[3]{\cos x}}dx$$

$$2I = \int_0^{\pi/2} \frac{\sqrt[3]{\cos x}+\sqrt[3]{\sin x}}{\sqrt[3]{\cos x}+\sqrt[3]{\sin x}}dx$$

$$2I = \int_0^{\pi/2} 1dx$$

$$2I = [x]_0^{\pi/2}$$

$$2I = \frac{\pi}{2}$$

$$\therefore I = \frac{\pi}{4}$$

Thank You

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