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312301 – Applied Mathematics (Sem II)

As per MSBTE's K Scheme

CO / CM / IF / AI / AN / DS

Unit I	Indefinite Integration	Marks - 20

Q. N	Solution
1	Evaluate: $\int x(x-1)^2 dx$ $\int x(x-1)^2 dx$ $= \int x(x^2-2x+1)\,dx$ $= \int (x^3-2x^2+x)\,dx$ $= \frac{x^4}{4} - \frac{2x^3}{3} + \frac{x^2}{2} + c$
2	Evaluate $\int 1/x^2+4\; dx$ $\int \frac{1}{x^2+4}\,dx$ $= \int \frac{1}{x^2+(2)^2}\,dx$ $= \frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right) + c$
3	Let I = $\int \frac{1}{\sqrt{9-4x^2}}\,dx$ $\therefore I = \int \frac{1}{\sqrt{4(\frac{9}{4}-x^2)}}\,dx$ $\therefore I = \frac{1}{\sqrt{4}}\int \frac{1}{\sqrt{\left(\frac{3}{2}\right)^2 - x^2}}\,dx$ $\therefore I = \frac{1}{2}\sin^{-1}\frac{x}{3/2} + c$
4	Evaluate $\int \sin^2 x\; dx$ $\int \sin^2 x\; dx$ $= \frac{1}{2}\int 2\sin^2 x\; dx$ $= \frac{1}{2}\int (1-\cos 2x)\,dx$ $= \frac{1}{2}\left(x - \frac{\sin 2x}{2}\right) + c$

5	Evaluate: $\int \cos^2 x dx$ $\int \cos^2 x dx = \int \frac{1+\cos 2x}{2} dx$ $= \frac{1}{2}\int (1+\cos 2x) dx$ $= \frac{1}{2}\left(x + \frac{\sin 2x}{2}\right) + c$
6	Evaluate $\int \sin^3 x dx$ $\int \sin^3 x dx$ $= \int \frac{3\sin x - \sin 3x}{4} dx$ $= \frac{1}{4}\left(3(-\cos x) - \left(\frac{-\cos 3x}{3}\right)\right) + c$ $= \frac{1}{4}\left(-3\cos x + \frac{\cos 3x}{3}\right) + c$
7	$Let\ I = \int \frac{sinx}{cos^2 x}\, dx$ $\therefore I = \int \frac{sinx}{cosx} \frac{1}{cosx} dx$ $\therefore I = \int tanx\ secx\, dx$ $\therefore I = secx + c$
8	Evaluate $\int x.e^x dx$ $\int x.e^x dx$ $= x\left(\int e^x dx\right) - \int\left(\int e^x dx \frac{d}{dx}(x)\right) dx$ $= xe^x - \int e^x.1\, dx$ $= xe^x - \int e^x\ dx$ $= xe^x - e^x + c$
9	Evaluate $\int \log x dx$ $\int \log x dx = \int \log x \cdot 1\, dx$ $= \log x \int 1dx - \int\left(\int 1dx\ \frac{d}{dx}\log x\right) dx$ $= \log x(x) - \int x\ \frac{1}{x} dx$ $= x\log x - \int 1dx$ $= x\log x - x + c$ $= x(\log x - 1) + c$

10	Evaluate: $\int x\cos x dx$ $\int x\cos x dx = x\int \cos x dx - \int\left(\int \cos x dx.\frac{d}{dx}x\right)dx$ $= x\sin x - \int(\sin x.1)dx$ $= x\sin x + \cos x + c$
11	Evaluate $\int \frac{1}{2x+5}dx$ $\int \frac{1}{2x+5}dx = \frac{1}{2}\left[\log\left(2x+5\right)\right]+c$
12	Evaluate: $\int \frac{1}{3x+5}dx$ $\int \frac{1}{3x+5}dx$ $=\frac{1}{3}\log(3x+5)+c$
13	Evaluate : $\int \frac{dx}{3x^2+4}$ $\int \frac{dx}{3x^2+4}$ $=\int \frac{dx}{\left(\sqrt{3}x\right)^2+2^2}$ $=\frac{1}{2}\tan^{-1}\left(\frac{\sqrt{3}x}{2}\right)\frac{1}{\sqrt{3}}+c$ $=\frac{1}{2\sqrt{3}}\tan^{-1}\left(\frac{\sqrt{3}x}{2}\right)+c$
14	$Let\ I = \int \frac{1}{3x+7}\ dx$ $\therefore I = \frac{log(3x+7)}{3} + c$
15	$Let\ I = \int e^{2x}\ dx$ $\therefore I = \frac{e^{2x}}{2} + c$

16	Evaluate $\int \frac{dx}{9+4x^2}$ $\int \frac{dx}{9+4x^2}$ $= \int \frac{dx}{3^2+(2x)^2}$ $= \frac{1}{3}\tan^{-1}\left(\frac{2x}{3}\right)\cdot\frac{1}{2}+c$ $= \frac{1}{6}\tan^{-1}\left(\frac{2x}{3}\right)+c$
17	$Let\ I = \int \frac{dx}{x(x-1)}$ $Consider, \frac{1}{x(x-1)} = \frac{A}{x} + \frac{B}{x-1}$ $\therefore 1 = A(x-1) + B(x)$ $Put\ x = 1, \quad B = 1$ $Put\ x = 0, \quad A = -1$ $\therefore \frac{1}{x(x-1)} = \frac{-1}{x} + \frac{1}{x-1}$ $\therefore I = \int \left(\frac{-1}{x} + \frac{1}{x-1}\right) dx$ $\therefore I = \int \frac{-1}{x} dx + \int \frac{1}{x-1} dx$ $\therefore I = -logx + log(x-1) + c$
18	Evaluate $\int \left[e^{2\log x} + e^{x\log 2}\right] dx$ $\int \left[e^{2\log x} + e^{x\log 2}\right] dx$ $= \int \left[e^{\log x^2} + e^{\log 2^x}\right] dx$ $= \int \left[x^2 + 2^x\right] dx$ $= \frac{x^3}{3} + \frac{2^x}{\log 2} + c$

19	Evaluate $e^{\int 2\log x\, dx}$ $e^{\int 2\log x\, dx}$ $= e^{2\int \log x\, dx}$ $= e^{2\int \log x.1\, dx}$ $= e^{2\left(\log x.x - \int x.\frac{1}{x}\, dx\right)}$ $= e^{2\left(x\log x - \int 1\, dx\right)}$ $= e^{2(x\log x - x)+c}$ $= e^{2x(\log x - 1)+c}$
20	$Let\ I = \int (x^2 - e^{3x})\, dx$ $\therefore I = \int x^2\, dx - \int e^{3x}\, dx$ $\therefore I = \frac{x^3}{3} - \frac{e^{3x}}{3} + c$
21	$\int \sqrt{1+\sin 2x}\, dx$ $= \int \sqrt{\sin^2 x + \cos^2 x + 2\sin x \cos x}\, dx$ $= \int \sqrt{(\sin x + \cos x)^2}\, dx$ $= \int (\sin x + \cos x)\, dx$ $= -\cos x + \sin x + c$
22	Evaluate $\int \frac{1}{1+\cos 2x}\, dx$ $\int \frac{1}{1+\cos 2x}\, dx$ $= \int \frac{1}{2\cos^2 x}\, dx\, dx$ $= \frac{1}{2} \int \sec^2 x\, dx$ $= \frac{1}{2} \tan x + c$
23	$Let\ I = \int sin^2 x\ \ cosx\, dx$ $Put\ sinx = t$ $cosx = \frac{dt}{dx}$ $cosx\, dx = -dt$ $\therefore I = \int t^2\, dt$

	$\therefore I = \frac{t^3}{3} + c$ $\therefore I = \frac{sin^3 x}{3} + c$
24	Evaluate $\int \frac{dx}{9x^2 - 16}$ $\int \frac{dx}{9x^2-16} = \frac{1}{9}\int \frac{dx}{x^2 - \frac{16}{9}}$ $= \frac{1}{9}\int \frac{dx}{x^2 - \left(\frac{4}{3}\right)^2}$ $= \frac{1}{9}\frac{1}{2.\frac{4}{3}}\log\left(\frac{x-\frac{4}{3}}{x+\frac{4}{3}}\right)+c$ $= \frac{1}{24}\log\left(\frac{3x-4}{3x+4}\right)+c$
25	$Let\ I = \int (e^x + x^e + e^e)\, dx$ $\therefore I = \int e^x\, dx + \int x^e\, dx + \int e^e\, dx$ $\therefore I = e^x + \frac{x^{e+1}}{e+1} + e^e x + c$
26	Evaluate $\int \left(x^a + a^x + a^a\right) dx$ $\int \left(x^a + a^x + a^a\right) dx$ $= \frac{x^{a+1}}{a+1} + \frac{a^x}{\log a} + a^a x + c$
27	Evaluate $\int \left[e^x + a^x + x^a + a^a\right] dx$ $\int \left[e^x + a^x + x^a + a^a\right] dx$ $= e^x + \frac{a^x}{\log a} + \frac{x^{a+1}}{a+1} + a^a x + c$

28	Evaluate $\int \frac{1-\cos 2x}{1+\cos 2x}\,dx$ $\int \frac{1-\cos 2x}{1+\cos 2x}\,dx$ $=\int \frac{2\sin^2 x}{2\cos^2 x}\,dx$ $=\int \tan^2 x\,dx$ $=\int (\sec^2 x-1)dx$ $=\tan x - x + c$
29	$Let\ I = \int cos^2x\,dx$ $\therefore I = \int \frac{1+cos2x}{2}dx$ $\therefore I = \frac{1}{2}\left[\int 1dx + \int cos2x\,dx\right]$ $\therefore I = \frac{1}{2}\left[x + \frac{sin2x}{2}\right] + c$
30	Evaluate $\int \tan^2 x dx$ $\int \tan^2 x dx$ $=\int (\sec^2 x-1)dx$ $=\tan x - x + c$
31	Evaluate $\int x^2 \cdot \log x dx$ $\int x^2\cdot\log x dx = \log x\int x^2 dx - \int\left[\int x^2 dx\cdot\frac{d}{dx}\log x\right]dx$ $=\log x\cdot\frac{x^3}{3} - \int \frac{x^3}{3}\cdot\frac{1}{x}dx$ $=\log x\cdot\frac{x^3}{3} - \frac{1}{3}\int x^2 dx$ $=\log x\cdot\frac{x^3}{3} - \frac{1}{3}\cdot\frac{x^3}{3} + c$ $=\log x\cdot\frac{x^3}{3} - \frac{x^3}{9} + c$
32	$Let\ I = \int \frac{(1+\sqrt{x})^2}{\sqrt{x}}dx$ $Put\ 1+\sqrt{x} = t$ $\therefore \frac{1}{2\sqrt{x}} = \frac{dt}{dx}$

	$\therefore \frac{1}{\sqrt{x}}dx = 2dt$ $\therefore I = \int t^2\,dt$ $\therefore I = \frac{t^3}{3} + c$ $\therefore I = \frac{(1+\sqrt{x})^3}{3} + c$
33	Evaluate : $\int \frac{1}{\sin^2 x \cos^2 x}\,dx$ $\int \frac{1}{\sin^2 x \cos^2 x}\,dx$ $= \int \frac{\sin^2 x + \cos^2 x}{\sin^2 x \cos^2 x}\,dx$ $= \int \frac{1}{\cos^2 x} + \frac{1}{\sin^2 x}\,dx$ $= \int \left(\sec^2 x + \cos ec^2 x\right)dx$ $= \tan x - \cot x + c$
34	Evaluate: $\int \frac{1}{\sqrt{1-x^2}\left(\sin^{-1} x\right)^2}\,dx$ $\int \frac{1}{\sqrt{1-x^2}\left(\sin^{-1} x\right)^2}\,dx$ Put $\sin^{-1} x = t$ $\therefore \frac{1}{\sqrt{1-x^2}}dx = dt$ $= \int \frac{1}{t^2}\,dt$ $= \int t^{-2}\,dt$ $= \frac{t^{-1}}{-1} + c$ $= -\left(\sin^{-1} x\right)^{-1} + c$

35	$\int \frac{e^{m\sin^{-1}x}}{\sqrt{1-x^2}}\,dx$ Put $\sin^{-1}x = t$ $\therefore \frac{1}{\sqrt{1-x^2}}dx = dt$ $= \int e^{mt}\,dt$ $= \frac{e^{mt}}{m} + c$ $= \frac{e^{m\sin^{-1}x}}{m} + c$
36	$Let\ I = \int \frac{cos(logx)}{x}\,dx$ $Put\ logx = t$ $\frac{1}{x}dx = dt$ $\therefore I = \int cost\,dt$ $\therefore I = sint + c$ $\therefore I = sin(logx) + c$
37	$\int \frac{e^x.\,dx}{(e^x-1)(e^x+1)}$ Put $e^x = t$ $\therefore e^x dx = dt$ $\therefore \int \frac{1}{(t-1)(t+1)}dt$ $= \int \frac{1}{t^2-1^2}dt$ $= \frac{1}{2(1)}\log\left(\frac{t-1}{t+1}\right)+c$ $= \frac{1}{2}\log\left(\frac{e^x-1}{e^x+1}\right)+c$
38	Evaluate $\int \frac{(x-1)e^x}{x^2.\sin^2\left(\frac{e^x}{x}\right)}dx$ $\int \frac{(x-1)e^x}{x^2.\sin^2\left(\frac{e^x}{x}\right)}dx$ Put $\frac{e^x}{x} = t$ $\therefore \frac{xe^x - e^x1}{x^2}\,dx = dt$

	$\therefore \frac{e^x(x-1)}{x^2}dx = dt$ $\int \frac{1}{\sin^2 t}dt$ $= \int \operatorname{cosec}^2 t\, dt$ $= -\cot t + c$ $= -\cot\left(\frac{e^x}{x}\right) + c$
39	Evaluate $\int \frac{1}{x+\sqrt{x}}dx$ $\int \frac{1}{x+\sqrt{x}}dx$ $= \int \frac{1}{\sqrt{x}\left(\sqrt{x}+1\right)}dx$ Put $\sqrt{x}+1 = t$ $\therefore \frac{1}{2\sqrt{x}}dx = dt$ $\therefore \frac{1}{\sqrt{x}}dx = 2dt$ $= 2\int \frac{1}{t}dt$ $= 2\log t + c$ $= 2\log\left(\sqrt{x}+1\right) + c$
40	Evaluate $\int \frac{e^x(x+1)}{\cos^2(xe^x)}dx$ $\int \frac{e^x(x+1)}{\cos^2(xe^x)}dx$ Put $xe^x = t$ $\therefore e^x(x+1)dx = dt$ $= \int \frac{1}{\cos^2 t}dt$ $= \int \sec^2 t\, dt$ $= \tan t + c$ $= \tan(xe^x) + c$

41	Evaluate $\int \frac{e^x(x+1)}{\sin^2(xe^x)}\,dx$ $\int \frac{e^x(x+1)}{\sin^2(xe^x)}\,dx$ Put $xe^x = t$ $\therefore e^x(x+1)dx = dt$ $\therefore \int \frac{1}{\sin^2 t}\,dt$ $= \int \operatorname{cosec}^2 t\,dt$ $= -\cot t + c$ $= -\cot(xe^x) + c$
42	$Let\ I = \int \frac{(tan^{-1}x)^3}{1+x^2}\,dx$ $Put\ \tan^{-1}x = t$ $\frac{1}{1+x^2}dx = dt$ $\therefore I = \int t^3\,dt$ $\therefore I = \frac{t^4}{4} + c$ $\therefore I = \frac{(\tan^{-1}x)^4}{4} + c$
43	$Let\ I = \int \frac{sec^2x}{3+tanx}dx$ $Put\ 3 + tanx = t$ $sec^2xdx = dt$ $\therefore I = \int \frac{1}{t}dt$ $\therefore I = logt + c$ $\therefore I = log(logx) + c$
44	$Let\ I = \int \frac{(\sin^{-1}x)^3}{\sqrt{1-x^2}}\,dx$ $Put\ \sin^{-1}x = t$ $\therefore \frac{1}{\sqrt{1-x^2}}dx = dt$ $\therefore I = \int t^3\,dt$ $\therefore I = \frac{t^4}{4} + c$ $\therefore I = \frac{(\sin^{-1}x)^4}{4} + c$

45	Evaluate: $\int \cos(\log x)\ dx$ $\int \cos(\log x)\ dx$ Put $\log x = t \Rightarrow x = e^t$ $\therefore \frac{1}{x}dx = dt$ $\therefore dx = xdt$ $\therefore dx = e^t dt$ $\therefore \int e^t \cos t dt$ $= \frac{e^t}{1+1}(1\cos t + 1\sin t) + c$ $= \frac{x}{2}(\cos(\log x) + \sin(\log x)) + c$
46	Evaluate: $\int \frac{\log\left(\tan x/2\right)}{\sin x}dx$ $\int \frac{\log\left(\tan x/2\right)}{\sin x}dx$ Put $\log\left(\tan x/2\right) = t$ $\frac{1}{\tan x/2}\sec^2 x/2\left(\frac{1}{2}\right)dx = dt$ $\left(\frac{1}{2}\right)\frac{\cos x/2}{\sin x/2}\cdot\frac{1}{\cos^2 x/2}dx = dt$ $\therefore \frac{1}{2\sin\frac{x}{2}\cos\frac{x}{2}}dx = dt$ $\therefore \frac{1}{\sin x}dx = dt$ $\therefore \int t\ dt$ $= \frac{t^2}{2} + c$ $= \frac{\left(\log\left(\tan x/2\right)\right)^2}{2} + c$

47	Evaluate: $\int \frac{\sec x \cos ecx}{\log \tan x} dx$ $\int \frac{\sec x \cos ecx}{\log \tan x} dx$ Put $\log \tan x = t$ $\therefore \frac{1}{\tan x} \sec^2 x dx = dt$ $\therefore \frac{\cos x}{\sin x} \frac{1}{\cos^2 x} dx = dt$ $\therefore \sec x \cos ecx dx = dt$ $= \int \frac{1}{t} dt$ $= \log t + c$ $= \log(\log(\tan x)) + c$
48	Evaluate : $\int \frac{1}{x\left[9+(\log_e x)^2\right]} dx$ $\int \frac{1}{x\left[9+(\log_e x)^2\right]} dx$ Let $\log_e x = t$ $\therefore \frac{1}{x} dx = dt$ $= \int \frac{1}{9+t^2} dt$ $= \int \frac{1}{3^2+t^2} dt$ $= \frac{1}{3} \tan^{-1}\left(\frac{t}{3}\right) + c$ $= \frac{1}{3} \tan^{-1}\left(\frac{\log_e x}{3}\right) + c$

49	Evaluate $\int \frac{x\sin^{-1}x}{\sqrt{1-x^2}}dx$ $\int \frac{x\sin^{-1}x}{\sqrt{1-x^2}}dx$ Let $\sin^{-1}x = t \quad \therefore x = \sin t$ $\therefore \frac{1}{\sqrt{1-x^2}}dx = dt$ $= \int t\sin t dt$ $= t\int \sin t dt - \int\left(\int \sin t dt \frac{d}{dt}t\right)dt$ $= t(-\cos t) - \int(-\cos t)dt$ $= -t\cos t + \sin t + c$ $= -\sin^{-1}x\cos\left(\sin^{-1}x\right) + x + c$
50	$Let\ I = \int \frac{1-tanx}{1+tanx}\ dx$ $\therefore I = \int \frac{1-\frac{sinx}{cosx}}{1+\frac{sinx}{cosx}}dx$ $\therefore I = \int \frac{cosx - sinx}{cosx + sinx}dx$ $Put\ cosx + sinx = t$ $\therefore -sinx + cosx = \frac{dt}{dx}$ $\therefore (cosx - sinx)dx = dt$ $\therefore I = \int \frac{1}{t}dt$ $\therefore I = logt + c$ $\therefore I = log(cosx + sinx) + c$
51	$Let\ I = \int \frac{cos\sqrt{x}}{\sqrt{x}}$ $Put\ \sqrt{x} = t$ $\therefore \frac{1}{2\sqrt{x}} = \frac{dt}{dx}$ $\therefore \frac{1}{\sqrt{x}}dx = 2dt$ $\therefore I = \int cost\ 2dt$ $\therefore I = 2\int cost\ dt$ $\therefore I = 2\ sint + c$ $\therefore I = 2\ sin\sqrt{x} + c$

52	Evaluate: $\int \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx$ $\int \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx$ Put $\sqrt{x}=t$ $\therefore \frac{1}{2\sqrt{x}}dx=dt$ $\therefore \frac{1}{\sqrt{x}}dx=2dt$ $=\int \sin t(2dt)$ $=-2\cos t+c$ $=-2\cos\sqrt{x}+c$
53	$Let\ I=\int \frac{1}{1-cos2x}\,dx$ $\therefore I=\int \frac{1}{2sin^2x}dx$ $\therefore I=\frac{1}{2}\int \frac{1}{sin^2x}dx$ $\therefore I=\frac{1}{2}\int cosec^2x\,dx$ $\therefore I=\frac{1}{2}(-cotx)+c$
54	Evaluate $\int \frac{\sec^2 x\,dx}{3\tan^2 x-2\tan x-5}$ $I=\int \frac{\sec^2 x\,dx}{3\tan^2 x-2\tan x-5}$ Put $\tan x=t$ $\therefore \sec^2 xdx=dt$ $=\int \frac{dt}{3t^2-2t-5}$ $=\frac{1}{3}\int \frac{dt}{t^2-\frac{2}{3}t-\frac{5}{3}}dt$ Third term $=\left(\frac{1}{2}\times\frac{-2}{3}\right)^2=\frac{1}{9}$ $=\frac{1}{3}\int \frac{dt}{t^2-\frac{2}{3}t+\frac{1}{9}-\frac{1}{9}-\frac{5}{3}}$ $=\ \frac{1}{3}\int \frac{dt}{\left(t-\frac{1}{3}\right)^2-\left(\frac{4}{3}\right)^2}$

	$=\frac{1}{3}\cdot\frac{1}{2\times\frac{4}{3}}\log\left(\frac{t-\frac{1}{3}-\frac{4}{3}}{t-\frac{1}{3}+\frac{4}{3}}\right)+c$ $=\frac{1}{8}\log\left(\frac{3t-5}{3t+3}\right)+c$ $I=\frac{1}{8}\log\left(\frac{3\tan x-5}{3\tan x+3}\right)+c$
55	Evaluate: $\int\frac{x}{(x^2+4)(x^2+9)}dx$ $\int\frac{x}{(x^2+4)(x^2+9)}dx$ Put $x^2=t$ $2xdx=dt$ $xdx=\frac{dt}{2}$ $\int\frac{\frac{dt}{2}}{(t+4)(t+9)}$ $=\frac{1}{2}\int\frac{dt}{(t+4)(t+9)}$ $\frac{1}{(t+4)(t+9)}=\frac{A}{t+4}+\frac{B}{t+9}$ $1=A(t+9)+B(t+4)$ Put $t=-4 \quad \therefore A=\frac{1}{5}$ Put $t=-9 \quad \therefore B=-\frac{1}{5}$ $\frac{1}{(t+4)(t+9)}=\frac{\frac{1}{5}}{t+4}+\frac{-\frac{1}{5}}{t+9}$ $\int\frac{dt}{(t+4)(t+9)}=\int\left(\frac{\frac{1}{5}}{t+4}+\frac{-\frac{1}{5}}{t+9}\right)dt$ $=\frac{1}{5}\log(t+4)-\frac{1}{5}\log(t+9)+c$ $=\frac{1}{5}\log(x^2+4)-\frac{1}{5}\log(x^2+9)+c$

56	Evaluate $\int \frac{\sec^2 x}{(1+\tan x)(3+\tan x)} dx$ $\int \frac{\sec^2 x}{(1+\tan x)(3+\tan x)} dx$ Put $\tan x = t$ $\therefore \sec^2 x dx = dt$ $\therefore \int \frac{1}{(1+t)(3+t)} dt$ $\frac{1}{(1+t)(3+t)} = \frac{A}{1+t} + \frac{B}{3+t}$ $\therefore 1 = A(3+t) + B(1+t)$ $\therefore$ Put $t=-1$, $A = \frac{1}{2}$ Put $t=-3$, $B = \frac{-1}{2}$ $\therefore \frac{1}{(1+t)(3+t)} = \frac{1/2}{1+t} - \frac{1/2}{3+t}$ $\therefore \int \frac{1}{(1+t)(3+t)} dt = \int \left(\frac{1/2}{1+t} - \frac{1/2}{3+t} \right) dt$ $= \frac{1}{2}\log(1+t) - \frac{1}{2}\log(3+t) + c$ $= \frac{1}{2}\log(1+\tan x) - \frac{1}{2}\log(3+\tan x) + c$
57	Evaluate : $\int \frac{\sec^2 x}{(1+\tan x)(2+\tan x)}$ $\int \frac{\sec^2 x}{(1+\tan x)(2+\tan x)} dx$ *Put* $\tan x = t$, $\therefore \sec^2 x dx = dt$ $\int \frac{1}{(1+t)(2+t)} dt$ $\frac{1}{(1+t)(2+t)} = \frac{A}{1+t} + \frac{B}{2+t}$ $1 = A(2+t) + B(1+t)$ $\therefore$ Put $t=-1$, $A=1$ Put $t=-2$, $B=-1$ $\therefore \frac{1}{(1+t)(2+t)} = \frac{1}{1+t} - \frac{1}{2+t}$ $\int \frac{1}{(1+t)(2+t)} dt = \int \left(\frac{1}{1+t} - \frac{1}{2+t} \right) dt$ $= \log[1+t] - \log[2+t] + c$ $= \log[1+\tan x] - \log[2+\tan x] + c$

58	Evaluate $\int \frac{\sec^2 x}{(1-\tan x)(2+\tan x)}dx$ $\int \frac{\sec^2 x}{(1-\tan x)(2+\tan x)}dx$ Let $\tan x = t$ $\therefore \sec^2 x dx = dt$ $= \int \frac{1}{(1-t)(2+t)}dt$ Cosider $\frac{1}{(1-t)(2+t)} = \frac{A}{1-t} + \frac{B}{2+t}$ $\therefore 1 = A(2+t) + B(1-t)$ Put $t = 1, \ \therefore A = \frac{1}{3}$ $t = -2, \therefore B = \frac{1}{3}$ $\therefore \frac{1}{(1-t)(2+t)} = \frac{\frac{1}{3}}{1-t} + \frac{\frac{1}{3}}{2+t}$ $\therefore \int \frac{1}{(1-t)(2+t)}dt = \int \left(\frac{\frac{1}{3}}{1-t} + \frac{\frac{1}{3}}{2+t} \right) dt$ $= \frac{1}{3}\frac{\log(1-t)}{-1} + \frac{1}{3}\log(2+t) + c$ $= \frac{-1}{3}\log(1-\tan x) + \frac{1}{3}\log(2+\tan x) + c$

59	Evaluate $\int \frac{\sec^2 x}{(1+\tan x)(2-\tan x)}dx$ $\int \frac{\sec^2 x}{(1+\tan x)(2-\tan x)}dx$ Put $\tan x = t$ $\therefore \sec^2 x\, dx = dt$ $\therefore \int \frac{1}{(1+t)(2-t)}dt$ $\frac{1}{(1+t)(2-t)} = \frac{A}{1+t} + \frac{B}{2-t}$ $\therefore 1 = A(2-t) + B(1+t)$ $\therefore$ Put $t=-1$, $A = \frac{1}{3}$ Put $t = 2$, $B = \frac{1}{3}$ $\therefore \frac{1}{(1+t)(2-t)} = \frac{1/3}{1+t} + \frac{1/3}{2-t}$ $\therefore \int \frac{1}{(1+t)(2-t)}dt = \int\left(\frac{1/3}{1+t} + \frac{1/3}{2-t}\right)dt$ $= \frac{1}{3}\log(1+t) + \frac{1}{3}\frac{\log(2-t)}{(-1)} + c$ $= \frac{1}{3}\log(1+\tan x) - \frac{1}{3}\log(2-\tan x) + c$

60	Evaluate: $\int \frac{\log x}{x(2+\log x)} \frac{dx}{(3+\log x)}$ $\int \frac{\log x}{x(2+\log x)(3+\log x)}\, dx$ Put $\log x = t$ $\therefore \frac{1}{x} dx = dt$ $\int \frac{t}{(2+t)(3+t)}\, dt$ consider $\frac{t}{(2+t)(3+t)} = \frac{A}{2+t} + \frac{B}{3+t}$ $\therefore t = A(3+t) + B(2+t)$ Put $t = -2$ $A = -2$ Put $t = -3$ $B = 3$ $\therefore \frac{t}{(2+t)(3+t)} = \frac{-2}{2+t} + \frac{3}{3+t}$ $\therefore \int \frac{t}{(2+t)(3+t)}\, dt = \int \left(\frac{-2}{2+t} + \frac{3}{3+t} \right) dt$ $= -2\log(2+t) + 3\log(3+t) + c$ $= -2\log(2+\log x) + 3\log(3+\log x) + c$

61	Evaluate : $\int \frac{1}{x(2-\log x)(2\log x-1)}dx$ $\int \frac{1}{x(2-\log x)(2\log x-1)}dx$ — Put $\log x = t$, $\therefore \frac{1}{x}dx = dt$ $\int \frac{1}{(2-t)(2t-1)}dt$ $\frac{1}{(2-t)(2t-1)} = \frac{A}{2-t} + \frac{B}{2t-1}$ $1 = A(2t-1) + B(2-t)$ $\therefore$ Put $t = 2$, $A = \frac{1}{3}$ Put $t = \frac{1}{2}$, $B = \frac{2}{3}$ $\therefore \frac{1}{(2-t)(2t-1)} = \frac{\frac{1}{3}}{2-t} + \frac{\frac{2}{3}}{2t-1}$ $\int \frac{1}{(2-t)(2t-1)}dt = \int \left(\frac{\frac{1}{3}}{2-t} + \frac{\frac{2}{3}}{2t-1} \right) dt$ $= -\frac{1}{3}\log[2-t] + \frac{2}{6}\log[2t-1] + c$ $= -\frac{1}{3}\log[2-\log x] + \frac{1}{3}\log[2\log x - 1] + c$
62	$Let\ I = \int \frac{cosx}{(4+sinx)(3+sinx)}\,dx$ $Put\ sinx = t$ $cosx\ dx = dt$ $\therefore I = \int \frac{1}{(4+t)(3+t)}dt$ $Consider, \frac{1}{(4+t)(3+t)} = \frac{A}{4+t} + \frac{B}{3+t}$ $\therefore 1 = A(3+t) + B(4+t)$ $Put\ x = -3, \quad B = 1$ $Put\ x = -4, \quad A = -1$ $\therefore \frac{1}{(4+t)(3+t)} = \frac{-1}{4+t} + \frac{1}{3+t}$ $\therefore I = \int \left[\frac{-1}{4+t} + \frac{1}{3+t}\right]dt$ $\therefore I = \int \frac{-1}{4+t}dt + \int \frac{1}{3+t}dt$ $\therefore I = -log(4+t) + log(3+t) + c$ $\therefore I = -log(4+sinx) + log(3+sinx) + c$
63	$Let\ I = \int \frac{cos\theta}{(2+sin\theta)(3+4sin\theta)}\,d\theta$ $Put\ sin\theta = t$ $cos\theta\ d\theta = dt$

	$\therefore I = \int \frac{1}{(2+t)(3+4t)} dt$ $Consider, \frac{1}{(2+t)(3+4t)} = \frac{A}{2+t} + \frac{B}{3+4t}$ $\therefore 1 = A(3+4t) + B(2+t)$ $Put\ x = -2, \quad A = -5$ $Put\ x = -3/4, \quad B = 5/4$ $\therefore \frac{1}{(2+t)(3+4t)} = \frac{-5}{2+t} + \frac{5/4}{3+4t}$ $\therefore I = \int \left[\frac{-5}{2+t} + \frac{5/4}{3+4t}\right] dt$ $\therefore I = -5\int \frac{1}{2+t} dt + \frac{5}{4}\int \frac{1}{3+4t} dt$ $\therefore I = -5\log(2+t) + \frac{5}{4}\frac{\log(3+4t)}{4} + c$ $\therefore I = -5\log(2+sin\theta) + \frac{5}{4}\frac{\log(3+4sin\theta)}{4} + c$ $\therefore I = -5\log(2+sin\theta) + 5\frac{\log(3+4sin\theta)}{16} + c$
64	$Let\ I = \int \frac{x}{x^2+3x-4}\ dx$ $\therefore I = \int \frac{x}{(x+4)(x-1)}\ dx$ $Consider, \frac{x}{(x+4)(x-1)} = \frac{A}{x+4} + \frac{B}{x-1}$ $\therefore x = A(x-1) + B(x+4)$ $Put\ x = 1,\ \ B = 1/5$ $Put\ x = -4,\ \ A = 4/5$ $\therefore \frac{x}{(x+4)(x-1)} = \frac{4/5}{x+4} + \frac{1/5}{x-1}$ $\therefore I = \int \left[\frac{4/5}{x+4} + \frac{1/5}{x-1}\right] dx$ $\therefore I = \int \frac{4/5}{x+4} dx + \int \frac{1/5}{x-1} dx$ $\therefore I = \frac{4}{5}\int \frac{1}{x+4} dx + \frac{1}{5}\int \frac{1}{x-1} dx$ $\therefore I = \frac{4}{5} log(x+4) + \frac{1}{5} log(x-1) + c$

65	Evaluate: $\int \frac{x+1}{x\left(x^2-4\right)} dx$ $\int \frac{x+1}{x\left(x^2-4\right)} dx = \int \frac{x+1}{x(x-2)(x+2)} dx$ Let $\frac{x+1}{x(x-2)(x+2)} = \frac{A}{x} + \frac{B}{x-2} + \frac{C}{x+2}$ $x+1 = A(x-2)(x+2) + Bx(x+2) + Cx(x-2)$ put $x=0 \quad \therefore A = \frac{-1}{4}$ put $x=2 \quad \therefore B = \frac{3}{8}$ put $x=-2 \quad \therefore C = \frac{-1}{8}$ $\frac{x+1}{x(x-2)(x+2)} = \frac{\frac{-1}{4}}{x} + \frac{\frac{3}{8}}{x-2} + \frac{\frac{-1}{8}}{x+2}$ $\int \frac{x+1}{x(x-2)(x+2)} dx = \int \left(\frac{\frac{-1}{4}}{x} + \frac{\frac{3}{8}}{x-2} + \frac{\frac{-1}{8}}{x+2} \right) dx$ $= \frac{-1}{4}\log x + \frac{3}{8}\log(x-2) - \frac{1}{8}\log(x+2) + c$
66	Evaluate $\int \frac{x}{(x+1)(x+2)} dx$ Consider $\frac{x}{(x+1)(x+2)} = \frac{A}{x+1} + \frac{B}{x+2}$ $\therefore x = A(x+2) + B(x+1)$ Put $x=-1$ $\therefore -1 = A(-1+2)$ $\therefore A = -1$ Put $x=-2$ $\therefore -2 = B(-2+1)$ $\therefore B = 2$ $\frac{x}{(x+1)(x+2)} = \frac{-1}{x+1} + \frac{2}{x+2}$ $\therefore \int \frac{x}{(x+1)(x+2)} dx = -\int \frac{1}{x+1} dx + 2\int \frac{1}{x+2} dx$ $= -\log(x+1) + 2\log(x+2) + c$

67	$\int\frac{2x^2+5}{(x-1)(x+2)(x+3)}dx$ Consider $\frac{2x^2+5}{(x-1)(x+2)(x+3)}=\frac{A}{x-1}+\frac{B}{x+2}+\frac{C}{x+3}$ $\therefore 2x^2+5=A(x+2)(x+3)+B(x-1)(x+3)+C(x-1)(x+2)$ Put $x=1\Rightarrow$ $2(1)^2+5=A(1+2)(1+3)$ $\therefore A=\frac{7}{12}$ Put $x=-2\Rightarrow$ $2(-2)^2+5=B(-2-1)(-2+3)$ $\therefore B=\frac{-13}{3}$ Put $x=-3\Rightarrow$ $2(-3)^2+5=C(-3-1)(-3+2)$ $\therefore C=\frac{23}{4}$ $\therefore\frac{2x^2+5}{(x-1)(x+2)(x+3)}=\frac{\frac{7}{12}}{x-1}+\frac{\frac{-13}{3}}{x+2}+\frac{\frac{23}{4}}{x+3}$ $\therefore\int\frac{2x^2+5}{(x-1)(x+2)(x+3)}dx=\int\left(\frac{\frac{7}{12}}{x-1}+\frac{\frac{-13}{3}}{x+2}+\frac{\frac{23}{4}}{x+3}\right)dx$ $=\frac{7}{12}\log(x-1)-\frac{13}{3}\log(x+2)+\frac{23}{4}\log(x+3)+c$
68	Evaluate $\int\frac{x^2+1}{(x+1)(x+2)(x-3)}dx$ $\frac{x^2+1}{(x+1)(x+2)(x-3)}=\frac{A}{x+1}+\frac{B}{x+2}+\frac{C}{x-3}$ $\therefore x^2+1=A(x+2)(x-3)+B(x+1)(x-3)+C(x+1)(x+2)$ Put $x=-1$ $\therefore A=\frac{-1}{2}$ Put $x=-2$ $\therefore B=1$ Put $x=3$ $\therefore C=\frac{1}{2}$ $\therefore\int\frac{x^2+1}{(x+1)(x+2)(x-3)}dx=\int\frac{\frac{-1}{2}}{x+1}+\frac{1}{x+2}+\frac{\frac{1}{2}}{x-3}dx$ $\therefore\int\frac{x^2+1}{(x+1)(x+2)(x-3)}dx=\frac{-1}{2}\log(x+1)+\log(x+2)+\frac{1}{2}\log(x-3)+c$

69	$Let\ I = \int \frac{dx}{x^2+3x+2}$ $\therefore I = \int \frac{1}{(x+2)(x+1)}\ dx$ $Consider, \frac{1}{(x+2)(x+1)} = \frac{A}{x+2} + \frac{B}{x+1}$ $\therefore 1 = A(x+1) + B(x+2)$ $Put\ x = -1, \quad B = 1$ $Put\ x = -2, \quad A = -1$ $\frac{1}{(x+2)(x+1)} = \frac{-1}{x+2} + \frac{1}{x+1}$ $\therefore I = \int \left[\frac{-1}{x+2} + \frac{1}{x+1}\right] dx$ $\therefore I = \int \frac{-1}{x+2} dx + \int \frac{1}{x+1} dx$ $\therefore I = -log(x+2) + log(x+1) + c$
70	Evaluate $\int \frac{dx}{x^2+4x+5}$ $\int \frac{dx}{x^2+4x+5}$ Third term $= \frac{(4x)^2}{4 \times x^2} = 4$ $= \int \frac{dx}{x^2+4x+4-4+5}$ $= \int \frac{dx}{(x+2)^2+1}$ $= \frac{1}{1} \tan^{-1}\left(\frac{x+2}{1}\right) + c$ $= \tan^{-1}(x+2) + c$
71	$Let\ I = \int \frac{1}{9x^2+6x+10} dx$ $\therefore I = \int \frac{1}{9(x^2+\frac{6}{9}x+\frac{10}{9})} dx$ $\therefore I = \frac{1}{9} \int \frac{1}{x^2+\frac{2}{3}x+\frac{10}{9}} dx$ $Third\ term = \left(\frac{1}{2} \times \frac{2}{3}\right)^2 = \frac{1}{9}$ $\therefore I = \frac{1}{9} \int \frac{1}{(x^2+\frac{2}{3}x+\frac{1}{9})+\frac{10}{9}-\frac{1}{9}} dx$ $\therefore I = \frac{1}{9} \int \frac{1}{\left(x+\frac{1}{3}\right)^2+1} dx$ $\therefore I = \frac{1}{9} \cdot \frac{1}{1/3}\ \tan^{-1}\left(\frac{x+3}{1/3}\right) + c$ $\therefore I = \frac{1}{3} \tan^{-1}\left(\frac{x+3}{1/3}\right) + c$

72	Evaluate: $\int \frac{x+1}{x^2(x-2)}dx$ $\int \frac{x+1}{x^2(x-2)}dx$ Consider $\frac{x+1}{x^2(x-2)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-2}$ $\therefore x+1 = Ax(x-2) + B(x-2) + Cx^2$ Put $x = 0$ $\therefore B = -\frac{1}{2}$ Put $x = 2$ $\therefore C = \frac{3}{4}$ Put $x = 1$ $2 = -A - B + C$ $\therefore 2 = -A + \frac{1}{2} + \frac{3}{4}$ $\therefore A = \frac{-3}{4}$ $\frac{x+1}{x^2(x-2)} = \frac{\frac{-3}{4}}{x} + \frac{\frac{-1}{2}}{x^2} + \frac{\frac{3}{4}}{x-2}$

73	Evaluate $\int x.\log(x+1)dx$ $\int x.\log(x+1)dx$ $=\log(x+1)\int xdx-\int\left(\int xdx.\frac{d}{dx}\log(x+1)\right)dx$ $=\log(x+1)\frac{x^2}{2}-\int\left(\frac{x^2}{2}\frac{1}{x+1}\right)dx$ $=\log(x+1)\frac{x^2}{2}-\frac{1}{2}\int\left(\frac{x^2}{x+1}\right)dx$ $x-1$ $x+1\overline{)x^2}$ $-\underline{x^2+x}$ $-x$ $\underline{-x-1}$ 1 $\frac{x^2}{x+1}=(x-1)+\frac{1}{x+1}$ $\therefore I=\log(x+1)\frac{x^2}{2}-\frac{1}{2}\int\left((x-1)+\frac{1}{x+1}\right)dx$ $\therefore I=\frac{1}{2}\left(\log(x+1)x^2-\left(\frac{x^2}{2}-x+\log(x+1)\right)\right)+c$
74	Evaluate $\int x\tan^{-1}x\,dx$ $\int\tan^{-1}x\cdot xdx$ $=\tan^{-1}x\int xdx-\int\left(\int xdx\frac{d}{dx}(\tan^{-1}x)\right)dx$ $=\tan^{-1}x\,\frac{x^2}{2}-\int\frac{x^2}{2}\frac{1}{1+x^2}dx$ $=\frac{x^2}{2}\tan^{-1}x-\frac{1}{2}\int\frac{x^2}{1+x^2}dx$ $=\frac{x^2}{2}\tan^{-1}x-\frac{1}{2}\int\left(\frac{1+x^2-1}{1+x^2}\right)dx$ $=\frac{x^2}{2}\tan^{-1}x-\frac{1}{2}\int\left(\frac{1+x^2}{1+x^2}-\frac{1}{1+x^2}\right)dx$ $=\frac{x^2}{2}\tan^{-1}x-\frac{1}{2}\int\left(1-\frac{1}{1+x^2}\right)dx$ $=\frac{x^2}{2}\tan^{-1}x-\frac{1}{2}(x-\tan^{-1}x)+c$
75	Evalute: $\int e^x.\sin 4xdx$ $\int e^x.\sin 4xdx$

	$= \sin 4x \int e^x dx - \int \left(\int e^x dx \frac{d}{dx} \sin 4x \right) dx$ $= \sin 4x e^x - \int \cos 4x \cdot 4 \cdot e^x dx$ $= \sin 4x e^x - 4\left[\cos 4x \int e^x dx - \int \left(\int e^x dx \cdot \frac{d}{dx} \cos 4x \right) dx \right]$ $= \sin 4x e^x - 4\left[\cos 4x e^x - \int \left(-\sin 4x \cdot 4 \cdot e^x \right) dx \right]$ $= \sin 4x e^x - 4\left[\cos 4x e^x + 4\int \sin 4x \cdot e^x dx \right]$ $= \sin 4x e^x - 4\cos 4x e^x - 16I$ $I + 16I = \sin 4x e^x - 4\cos 4x e^x$ $17I = \sin 4x e^x - 4\cos 4x e^x$ $I = \frac{1}{17}\left(\sin 4x e^x - 4\cos 4x e^x \right)$
76	Evaluate $\int x^2 . e^{3x} \, dx$ $\int x^2 . e^{3x} \, dx$ $= x^2 \left(\int e^{3x} dx \right) - \int \left(\int e^{3x} dx . \frac{d}{dx}(x^2) \right) dx$ $= x^2 \frac{e^{3x}}{3} - \int \frac{e^{3x}}{3} .2x \, dx$ $= \frac{x^2 e^{3x}}{3} - \frac{2}{3} \int x e^{3x} \, dx$ $= \frac{x^2 e^{3x}}{3} - \frac{2}{3}\left[x\left(\int e^{3x} dx \right) - \int \left(\int e^{3x} dx . \frac{d}{dx}(x) \right) dx \right]$ $= \frac{x^2 e^{3x}}{3} - \frac{2}{3}\left[x\frac{e^{3x}}{3} - \int \frac{e^{3x}}{3} .1 \, dx \right]$ $= \frac{x^2 e^{3x}}{3} - \frac{2}{3}\left[\frac{x e^{3x}}{3} - \frac{e^{3x}}{9} \right] + c$
77	Evaluate $\int \tan^{-1} x \, dx$ $\int \tan^{-1} x \, dx = \int \tan^{-1} x \, .1 \, dx$ $= \tan^{-1} x \int 1 dx - \int \left(\left(\int 1 dx \right) \frac{d}{dx}\left(\tan^{-1} x \right) dx \right)$ $= x \tan^{-1} x - \int x \frac{1}{1+x^2} dx$ $= x \tan^{-1} x - \int \frac{x}{1+x^2} dx$ $= x \tan^{-1} x - \frac{1}{2} \int \frac{2x}{1+x^2} dx$ $= x \tan^{-1} x - \frac{1}{2} \log\left(1+x^2 \right) + c$

78	$Let\ I = \int sin^{-1}x\,dx$ $\therefore I = \int 1.sin^{-1}x\,dx$ $\therefore By\ LIATE\ rule,\quad u = \sin^{-1}x \ \ \&\ \ v = 1$ $\therefore By\ integration\ by\ parts\ formula,$ $\therefore I = \sin^{-1}x \int 1\,dx - \int \left[\frac{d(\sin^{-1}x)}{dx} \times \int 1\,dx\right] dx$ $\therefore I = \sin^{-1}x\,.\,x - \int \frac{1}{\sqrt{1-x^2}}\,.\,x\,dx$ $\therefore I = \sin^{-1}x\,.\,x - \int \frac{-2x}{-2\sqrt{1-x^2}} dx$ $\therefore I = \sin^{-1}x\,.\,x + \frac{1}{2}\int \frac{-2x}{\sqrt{1-x^2}} dx$ $\therefore I = \sin^{-1}x\,.\,x + \frac{1}{2}2\sqrt{1-x^2} + c$ $\therefore I = \sin^{-1}x\,.\,x + \sqrt{1-x^2} + c$
79	$\int \frac{dx}{\sqrt{13-6x-x^2}}$ $\int \frac{dx}{\sqrt{13-6x-x^2}}$ $\text{Third term} = \left(\frac{1}{2}\times -6\right)^2 = 9$ $= \int \frac{dx}{\sqrt{13+9-9-6x-x^2}}$ $= \int \frac{dx}{\sqrt{22-(x+3)^2}}$ $= \int \frac{dx}{\sqrt{\sqrt{22}^2-(x+3)^2}}$ $= \sin^{-1}\left(\frac{x+3}{\sqrt{22}}\right) + c$

80	Evaluate $\int \frac{1}{\sqrt{16-6x-x^2}}dx$ $\int \frac{1}{\sqrt{16-6x-x^2}}dx$ $\text{Third Term} = \frac{(6)^2}{4} = 9$ $= \int \frac{1}{\sqrt{16+9-9-6x-x^2}}dx$ $= \int \frac{1}{\sqrt{25-(9+6x+x^2)}}dx$ $= \int \frac{1}{\sqrt{(5)^2-(x+3)^2}}dx$ $= \sin^{-1}\left(\frac{x+3}{5}\right)+c$
81	$Let\ I = \int \frac{1}{\sqrt{3-x-x^2}}\ dx$ $Third\ term = \left(\frac{1}{2}\times(-1)\right)^2 = \frac{1}{4}$ $\therefore I = \int \frac{1}{\sqrt{-(-3+x+x^2)}}\ dx$ $\therefore I = \int \frac{1}{\sqrt{-(x^2+x-3+\frac{1}{4}-\frac{1}{4})}}\ dx$ $\therefore I = \int \frac{1}{\sqrt{-[(x^2+x+\frac{1}{4})-3-\frac{1}{4}]}}\ dx$ $\therefore I = \int \frac{1}{\sqrt{-[\left(x+\frac{1}{2}\right)^2-\frac{13}{4}]}}\ dx$ $\therefore I = \int \frac{1}{\sqrt{-\left(x+\frac{1}{2}\right)^2+\frac{13}{4}}}\ dx$ $\therefore I = \int \frac{1}{\sqrt{\frac{13}{4}-\left(x+\frac{1}{2}\right)^2}}\ dx$ $\therefore I = \int \frac{1}{\sqrt{\left(\frac{\sqrt{13}}{2}\right)^2-\left(x+\frac{1}{2}\right)^2}}\ dx$ $\therefore I = \sin^{-1}\left(\frac{x+\frac{1}{2}}{\sqrt{13}/2}\right)+c$

82	Evaluate $\int \frac{1}{\sqrt{x^2+4x+13}}dx$ $\int \frac{dx}{\sqrt{x^2+4x+13}}$ Third term $= \frac{(4)^2}{4} = 4$ $= \int \frac{dx}{\sqrt{x^2+4x+4+13-4}}$ $= \int \frac{dx}{\sqrt{(x+2)^2+9}}$ $= \int \frac{dx}{\sqrt{(x+2)^2+3^2}}$ $= \log\left((x+2)+\sqrt{(x+2)^2+3^2}\right)+c$
83	$\int \frac{1}{2x^2+3x+1}dx = \int \frac{1}{(2x+1)(x+1)}dx$ Let $\frac{1}{(2x+1)(x+1)} = \frac{A}{2x+1} + \frac{B}{x+1}$ $1 = A(x+1) + B(2x+1)$ Put $x = \frac{-1}{2}$ $\therefore A = 2$ Put $x = -1$ $\therefore B = -1$ $\frac{1}{(2x+1)(x+1)} = \frac{2}{2x+1} + \frac{-1}{x+1}$ $\int \frac{1}{(2x+1)(x+1)}dx = \int\left(\frac{2}{2x+1} + \frac{-1}{x+1}\right)dx$ $= \frac{2\log(2x+1)}{2} - \log(x+1) + c$ $= \log(2x+1) - \log(x+1) + c$
84	$Let\ I = \int \frac{dx}{3+2x-x^2}$ $Third\ term = \left(\frac{1}{2}\times 2\right)^2 = 1$ $\therefore I = \int \frac{1}{-(-3-2x+x^2)}dx$ $\therefore I = \int \frac{1}{-(x^2-2x-3+1-1)}dx$ $\therefore I = \int \frac{1}{-[(x^2-2x+1)-3-1]}dx$ $\therefore I = \int \frac{1}{[(x-1)^2-4]}dx$

	$\therefore I = \int \frac{1}{[(x-1)^2 - 2^2]} dx$ $\therefore I = \frac{1}{2 \times 2} \log \left\| \frac{(x-1)-2}{(x-1)+2} \right\| + c$ $\therefore I = \frac{1}{4} \log \left\| \frac{x-3}{x+1} \right\| + c$
85	Evaluate : $\int \sec^3 x\, dx$ Let $I = \int \sec^3 x\, dx$ $= \int \sec^2 x \cdot \sec x\, dx$ $= \sec x \int \sec^2 x\, dx - \int \left[\int \sec^2 x\, dx \cdot \frac{d}{dx} \sec x \right] dx$ $= \sec x \tan x - \int [\tan x \cdot \sec x \cdot \tan x] dx$ $= \sec x \tan x - \int \tan^2 x \cdot \sec x dx$ $= \sec x \tan x - \int (\sec^2 x - 1) \cdot \sec x dx$ $= \sec x \tan x - \int (\sec^3 x - \sec x) dx$ $= \sec x \tan x - \int \sec^3 x dx + \int \sec x dx$ $I = \sec x \tan x - I + \log(\sec x + \tan x) + c$ $\therefore 2I = \sec x \tan x + \log(\sec x + \tan x) + c$ $\therefore I = \frac{1}{2}(\sec x \tan x + \log(\sec x + \tan x)) + c$
86	Evaluate : $\int \frac{1}{x^2 + 4x + 9} dx$ $\int \frac{1}{x^2 + 4x + 9} dx$ Third term $= \left(\frac{1}{2} \times 4 \right)^2 = 4$ $= \int \frac{1}{x^2 + 4x + 4 - 4 + 9} dx$ $= \int \frac{1}{(x+2)^2 + (\sqrt{5})^2} dx$ $= \frac{1}{\sqrt{5}} \tan^{-1} \left(\frac{x+2}{\sqrt{5}} \right) + c$

87	Evaluate : $\int \frac{dx}{x^2+4x+25}$ $I=\int \frac{dx}{x^2+4x+25}$ $T.T.=\left(\frac{1}{2}\times Coeff.of\ x\right)^2=\left(\frac{1}{2}\times 4\right)^2=4$ $x^2+4x+25=x^2+4x+4-4+25$ $=(x+2)^2+21=(x+2)^2+\left(\sqrt{21}\right)^2$ $\therefore I=\int \frac{dx}{(x+2)^2+\left(\sqrt{21}\right)^2}$ $=\frac{1}{\sqrt{21}}\tan^{-1}\left(\frac{x+2}{\sqrt{21}}\right)+c$
88	Evaluate : $\int \frac{\cos x}{1+\sin^2 x}dx$ Put $\sin x=t$ $\therefore \cos x dx=dt$ $=\int \frac{dt}{1+t^2}$ $=\tan^{-1}t+c$ $=\tan^{-1}(\sin x)+c$
89	Evaluate $\int x^2\cos 2x\,dx$ $I=\int x^2\cos 2x\,dx$ $=x^2\cdot\int \cos 2x\,dx-\int\left[\int \cos 2x\,dx\frac{d}{dx}(x^2)\right]dx$ $=x^2\frac{\sin 2x}{2}-\int \frac{\sin 2x}{2}(2x)\,dx$ $=2x^2\frac{\sin 2x}{2}-\int x\sin 2x\,dx$ $=x^2(\sin 2x)-\left[x\int(\sin 2x)\,dx-\int\left(\int(\sin 2x)\,dx\frac{d}{dx}(x)\right)\right]dx$ $=x^2\sin 2x-x\left(-\frac{\cos 2x}{2}\right)+\int\left(-\frac{\cos 2x}{2}\right)1dx$ $=x^2\sin 2x+\frac{x}{2}\cos 2x-\frac{1}{2}\int \cos 2x\,dx$ $=x^2\sin 2x+\frac{x}{2}\cos 2x-\frac{1}{4}\sin 2x+c$

90	Evaluate $\int x^2.\tan x\ dx$ $\int x^2.\tan x\ dx$ $= x^2\left(\int \tan x dx\right) - \int\left(\int \tan x dx.\frac{d}{dx}(x^2)\right)dx$ $= x^2 \log(\sec x) - \int \log(\sec x).2x\ dx$ $= x^2 \log(\sec x) - 2\left[\log(\sec x)\frac{x^2}{2} - \int \frac{1}{\sec x}.\sec x \tan x.\frac{x^2}{2}dx\right]$ $= x^2 \log(\sec x) - 2\left[\log(\sec x)\frac{x^2}{2} - \frac{1}{2}\int x^2.\tan x dx\right]$ $= x^2 \log(\sec x) - 2\left[\log(\sec x)\frac{x^2}{2} - \frac{1}{2}I\right]$ $I = x^2 \log(\sec x) - \log(\sec x)x^2 + I$
91	Evaluate : $\int x \operatorname{cosec}^{-1} x dx$ $\int x \operatorname{cosec}^{-1} x dx$ $= \operatorname{cosec}^{-1} x \int x dx - \int\left(\int x dx\ \frac{d}{dx}\operatorname{cosec}^{-1} x\right)dx$ $= \operatorname{cosec}^{-1} x \cdot \frac{x^2}{2} - \int \frac{x^2}{2}\cdot\left(\frac{-1}{x\sqrt{x^2-1}}\right)\cdot dx$ $= \operatorname{cosec}^{-1} x \cdot \frac{x^2}{2} + \frac{1}{2}\int \frac{x}{\sqrt{x^2-1}}\cdot dx$ $= \operatorname{cosec}^{-1} x \cdot \frac{x^2}{2} + \frac{1}{4}\int \frac{2x}{\sqrt{x^2-1}}\cdot dx$ $= \operatorname{cosec}^{-1} x \cdot \frac{x^2}{2} + \frac{1}{4}\left(2\sqrt{x^2-1}\right) + c$ $= \operatorname{cosec}^{-1} x \cdot \frac{x^2}{2} + \frac{1}{2}\left(\sqrt{x^2-1}\right) + c$
92	*Let* $I = \int x \log x\ dx$ *By LIATE rule,* $u = \log x\ \ \&\ v = x$ $\therefore I = \log x \int x\,dx - \int [\frac{d(\log x)}{dx} \times \int x\,dx]\,dx$ $\therefore I = \log x \frac{x^2}{2} - \int [\frac{1}{x} \times \frac{x^2}{2}]\,dx$ $\therefore I = \log x \frac{x^2}{2} - \frac{1}{2}\int x\,dx$ $\therefore I = \log x \frac{x^2}{2} - \frac{1}{2}\frac{x^2}{2} + c$ $\therefore I = \log x \frac{x^2}{2} - \frac{x^2}{4} + c$

93	$Let\ I = \int \frac{1}{3+2sinx} dx$ $Put\ tan\left(\frac{x}{2}\right) = t, \quad dx = \frac{2dt}{1+t^2}, \quad sinx = \frac{2t}{1+t^2}$ $\therefore I = \int \frac{\left(\frac{2dt}{1+t^2}\right)}{3+2.\frac{2t}{1+t^2}}$ $\therefore I = \int \frac{\left(\frac{2dt}{1+t^2}\right)}{\frac{3(1+t^2)+4t}{1+t^2}}$ $\therefore I = \int \frac{2\ dt}{3+3t^2+4t} dt$ $\therefore I = 2\int \frac{1}{3(1+t^2+\frac{4}{3}t)} dt$ $\therefore I = \frac{2}{3}\int \frac{1}{t^2+\frac{4}{3}t+1} dt$ $Third\ term = \left(\frac{1}{2}\times\frac{4}{3}\right)^2 = \frac{4}{9}$ $\therefore I = \frac{2}{3}\int \frac{1}{t^2+\frac{4}{3}t+1+\frac{4}{9}-\frac{4}{9}} dt$ $\therefore I = \frac{2}{3}\int \frac{1}{(t^2+\frac{4}{3}t+\frac{4}{9})+1-\frac{4}{9}} dt$ $\therefore I = \frac{2}{3}\int \frac{1}{\left(t+\frac{2}{3}\right)^2+\frac{5}{9}} dt$ $\therefore I = \frac{2}{3}\int \frac{1}{\left(t+\frac{2}{3}\right)^2+\left(\frac{\sqrt{5}}{3}\right)^2} dt$ $\therefore I = \frac{2}{3}.\frac{1}{\sqrt{5}/3}\ \tan^{-1}\left(\frac{t+\frac{2}{3}}{\sqrt{5}/3}\right)+c$ $\therefore I = \frac{2}{\sqrt{5}}.\ \tan^{-1}\left(\frac{tan\frac{x}{2}+\frac{2}{3}}{\sqrt{5}/3}\right)+c$
94	Evaluate: $\int \frac{dx}{3-2\sin x}$ $\int \frac{dx}{3-2\sin x}$ Put $\tan\frac{x}{2} = t \quad , dx = \frac{2dt}{1+t^2} \quad , \sin x = \frac{2t}{1+t^2}$ $\int \frac{\frac{2dt}{1+t^2}}{3-2\left(\frac{2t}{1+t^2}\right)}$

	$= 2\int \frac{dt}{3(1+t^2)-2(2t)}$ $= 2\int \frac{dt}{3+3t^2-4t}$ $= 2\int \frac{dt}{3t^2-4t+3}$ $= \frac{2}{3}\int \frac{dt}{t^2-\frac{4}{3}t+1}$ $T.T. = \left(\frac{1}{2}\times\frac{4}{3}\right)^2 = \frac{4}{9}$ $= \frac{2}{3}\int \frac{dt}{t^2-\frac{4}{3}t+\frac{4}{9}-\frac{4}{9}+1}$ $= \frac{2}{3}\int \frac{dt}{\left(t-\frac{2}{3}\right)^2+\frac{5}{9}}$ $= \frac{2}{3}\int \frac{dt}{\left(t-\frac{2}{3}\right)^2+\left(\frac{\sqrt{5}}{3}\right)^2}$ $= \frac{2}{3}\frac{1}{\frac{\sqrt{5}}{3}}\tan^{-1}\left(\frac{t-\frac{2}{3}}{\frac{\sqrt{5}}{3}}\right)+c$ $= \frac{2}{\sqrt{5}}\tan^{-1}\left(\frac{\tan\left(\frac{x}{2}\right)-\frac{2}{3}}{\frac{\sqrt{5}}{3}}\right)+c$
95	Evaluate : $\int \frac{1}{2+3\cos x}dx$ $\int \frac{1}{2+3\cos x}dx$ Put $\tan\frac{x}{2}=t \quad \cos x=\frac{1-t^2}{1+t^2}, dx=\frac{2dt}{1+t^2}$ $\therefore \int \frac{dx}{2+3\cos x}=\int \frac{1}{2+3\left(\frac{1-t^2}{1+t^2}\right)}\cdot\frac{2dt}{1+t^2}$ $=2\int \frac{1}{5-t^2}dt$ $=2\int \frac{1}{\left(\sqrt{5}\right)^2-t^2}dt$ $=2\times\frac{1}{2\sqrt{5}}\log\left(\frac{\sqrt{5}+t}{\sqrt{5}-t}\right)+c$ $=\frac{1}{\sqrt{5}}\log\left(\frac{\sqrt{5}+\tan\frac{x}{2}}{\sqrt{5}-\tan\frac{x}{2}}\right)+c$

96

Evaluate : $\int \frac{1}{5+4\cos x}dx$

$$\int \frac{1}{5+4\cos x}dx$$

Put $\tan\frac{x}{2}=t \quad \therefore \cos x=\frac{1-t^2}{1+t^2}, \quad dx=\frac{2dt}{1+t^2}$

$$\therefore \int \frac{dx}{5+4\cos x}=\int \frac{1}{5+4\left(\frac{1-t^2}{1+t^2}\right)}\cdot\frac{2dt}{1+t^2}$$

$$=2\int \frac{1}{t^2+9}dt$$

$$=2\int \frac{1}{t^2+3^2}dt$$

$$=2\times\frac{1}{3}\tan^{-1}\left(\frac{t}{3}\right)+c$$

$$=\frac{2}{3}\tan^{-1}\left(\frac{\tan\frac{x}{2}}{3}\right)+c$$

97

Evaluate $\int \frac{dx}{4+5\cos x}$

$$\int \frac{dx}{4+5\cos x}$$

Put $\tan\frac{x}{2}=t$, $dx=\frac{2dt}{1+t^2}$, $\cos x=\frac{1-t^2}{1+t^2}$

$$=\int \frac{\frac{2dt}{1+t^2}}{4+5\left(\frac{1-t^2}{1+t^2}\right)}$$

$$=\int \frac{2dt}{4(1+t^2)+5(1-t^2)}$$

$$=2\int \frac{dt}{4+4t^2+5-5t^2}$$

$$=2\int \frac{dt}{9-t^2}$$

$$=2\int \frac{dt}{(3)^2-t^2}$$

$$=2\frac{1}{2(3)}\log\left|\frac{3+t}{3-t}\right|+c$$

$$=\frac{1}{3}\log\left|\frac{3+\tan\frac{x}{2}}{3-\tan\frac{x}{2}}\right|+c$$

98	Evaluate $\int \frac{dx}{5+3\cos 2x}$ $\int \frac{dx}{5+3\cos 2x}$ Put $\tan x = t$, $dx = \frac{dt}{1+t^2}$ $\cos 2x = \frac{1-t^2}{1+t^2}$ $\int \frac{\frac{dt}{1+t^2}}{5+3\left(\frac{1-t^2}{1+t^2}\right)}$ $= \int \frac{dt}{5(1+t^2)+3(1-t^2)}$ $= \int \frac{dt}{5+5t^2+3-3t^2}$ $= \int \frac{dt}{2t^2+8}$ $= \int \frac{dt}{(\sqrt{2}t)^2+(\sqrt{8})^2}$ OR $= \frac{1}{2}\int \frac{dt}{t^2+4}$ $= \frac{1}{\sqrt{8}}\tan^{-1}\left(\frac{\sqrt{2}t}{\sqrt{8}}\right)\cdot\frac{1}{\sqrt{2}}+c$ $= \frac{1}{2}\cdot\frac{1}{2}\tan^{-1}\left(\frac{t}{2}\right)+c$ $= \frac{1}{\sqrt{16}}\tan^{-1}\left(\frac{\sqrt{2}\tan x}{\sqrt{8}}\right)+c$ OR $= \frac{1}{4}\tan^{-1}\left(\frac{t}{2}\right)+c$ $= \frac{1}{4}\tan^{-1}\left(\frac{\tan x}{2}\right)+c$ OR $= \frac{1}{4}\tan^{-1}\left(\frac{\tan x}{2}\right)+c$
99	Evaluate: $\int \frac{1}{2\sin x+3\cos x}dx$ $\int \frac{1}{2\sin x+3\cos x}dx$ Let $\tan\frac{x}{2} = t$ $\therefore \sin x = \frac{2t}{1+t^2}, \cos x = \frac{1-t^2}{1+t^2}, dx = \frac{2dt}{1+t^2}$ $= \int \frac{1}{2\left(\frac{2t}{1+t^2}\right)+3\left(\frac{1-t^2}{1+t^2}\right)}\cdot\frac{2dt}{1+t^2}$ $= \int \frac{1}{4t+3-3t^2}\cdot 2dt$ $= \int \frac{1}{-(3t^2-4t-3)}\cdot 2dt$ Third term $= \frac{(-4t)^2}{4\times 3t^2} = \frac{4}{3}$

	$=-2\int \frac{1}{3t^2-4t+\frac{4}{3}-\frac{4}{3}-3}dt$ $=-2\int \frac{1}{\left(\sqrt{3}t-\frac{2}{\sqrt{3}}\right)^2-\left(\sqrt{\frac{13}{3}}\right)^2}dt$ $=-2\cdot\frac{1}{2\sqrt{\frac{13}{3}}}\log\left(\frac{\sqrt{3}t-\frac{2}{\sqrt{3}}-\sqrt{\frac{13}{3}}}{\sqrt{3}t-\frac{2}{\sqrt{3}}+\sqrt{\frac{13}{3}}}\right)\cdot\frac{1}{\sqrt{3}}+c$ $=\frac{-1}{\sqrt{13}}\log\left(\frac{\sqrt{3}\tan\frac{x}{2}-\frac{2}{\sqrt{3}}-\sqrt{\frac{13}{3}}}{\sqrt{3}\tan\frac{x}{2}-\frac{2}{\sqrt{3}}+\sqrt{\frac{13}{3}}}\right)+c$ $=\frac{-1}{\sqrt{13}}\log\left(\frac{3\tan\frac{x}{2}-2-\sqrt{13}}{3\tan\frac{x}{2}-2+\sqrt{13}}\right)+c$
100	Evaluate $\int\frac{dx}{1+\sin x+\cos x}$ $\int\frac{dx}{1+\sin x+\cos x}$ Put $\tan\frac{x}{2}=t$ $\quad\therefore \sin x=\frac{2t}{1+t^2}$ $,\cos x=\frac{1-t^2}{1+t^2},\quad dx=\frac{2dt}{1+t^2}$ $\therefore\int\frac{dx}{1+\sin x+\cos x}=\int\frac{1}{1+\frac{2t}{1+t^2}+\left(\frac{1-t^2}{1+t^2}\right)}\cdot\frac{2dt}{1+t^2}$ $=\int\frac{2\,dt}{1+t^2+2t+1-t^2}dt$ $=2\int\frac{1}{2t+2}dt$ $=\int\frac{dt}{t+1}$ $=\log(t+1)+c$ $=\log\left(\tan\frac{x}{2}+1\right)+c$
101	$Let\ I=\int\frac{dx}{3+2sinx+\ cosx}$ $Put\ tan\left(\frac{x}{2}\right)=t,\quad dx=\frac{2dt}{1+t^2},\quad sinx=\frac{2t}{1+t^2}\ ,cosx=\frac{1-t^2}{1+t^2}$ $\therefore I=\int\frac{\frac{2dt}{1+t^2}}{3+2\frac{2t}{1+t^2}+\frac{1-t^2}{1+t^2}}$ $\therefore I=\int\frac{2dt}{3(1+t^2)+4t+1-t^2}$ $\therefore I=2\int\frac{dt}{3+3t^2+4t+1-t^2}$ $\therefore I=2\int\frac{dt}{2t^2+4t+4}$ $\therefore I=2\int\frac{dt}{2(t^2+2t+2)}$

	$\therefore I = \int \frac{dt}{(t^2+2t+2)}$ $Third\ term = \left(\frac{1}{2}\times 2\right)^2 = 1$ $\therefore I = \int \frac{1}{t^2+2t+2+1-1}dt$ $\therefore I = \int \frac{1}{(t^2+2t+1)+2-1}dt$ $\therefore I = \int \frac{1}{(t+1)^1+1}dt$ $\therefore I = \tan^{-1}\left(\frac{t+1}{1}\right)+c$ $\therefore I = \tan^{-1}\left(tan\frac{x}{2}+1\right)+c$
102	Evaluate $\int \frac{dx}{5-4\sin x}$ $\int \frac{dx}{5-4\sin x}$ Let $\tan\left(\frac{x}{2}\right)=t$ $\therefore \sin x = \frac{2t}{1+t^2}, \quad dx = \frac{2dt}{1+t^2}$ $= \int \frac{1}{5-4\left(\frac{2t}{1+t^2}\right)}\frac{2dt}{1+t^2}$ $= \int \frac{1}{5\left(1+t^2\right)-8t}2dt$

	$=2\int \frac{1}{5t^2-8t+5}dt$ $=2\int \frac{1}{5\left(t^2-\frac{8}{5}t+1\right)}dt$ Third term $=\left(\frac{1}{2}\times \text{coefficient of } t\right)^2$ $=\left(\frac{1}{2}\times\frac{-8}{5}\right)^2$ $=\frac{16}{25}$ $=2\int \frac{1}{5\left(t^2-\frac{8}{5}t+\frac{16}{25}-\frac{16}{25}+1\right)}dt$ $=\frac{2}{5}\int \frac{1}{\left(t-\frac{4}{5}\right)^2+\frac{9}{25}}dt$ $=\frac{2}{5}\int \frac{1}{\left(t-\frac{4}{5}\right)^2+\left(\frac{3}{5}\right)^2}dt$ $=\frac{2}{5}\cdot\frac{1}{\frac{3}{5}}\tan^{-1}\left(\frac{t-\frac{4}{5}}{\frac{3}{5}}\right)+c$ $=\frac{2}{3}\tan^{-1}\left(\frac{5t-4}{3}\right)+c$ $=\frac{2}{3}\tan^{-1}\left(\frac{5\tan\frac{x}{2}-4}{3}\right)+c$
103	Evaluate: $\int \frac{x}{1+\cos 2x}dx$ $\int \frac{x}{1+\cos 2x}dx$ $=\int \frac{x}{2\cos^2 x}dx$ $=\frac{1}{2}\int x\sec^2 x dx$ $=\frac{1}{2}\left[x\int \sec^2 x dx-\int\left(\int \sec^2 x dx.\frac{d}{dx}x\right)dx\right]$ $=\frac{1}{2}\left[x\tan x-\int \tan x.1dx\right]$ $=\frac{1}{2}\left[x\tan x-\log\left(\sec x\right)\right]+c$

104	Evaluate $\int \frac{dx}{4\cos^2 x+9\sin^2 x}$ $\int \frac{dx}{4\cos^2 x+9\sin^2 x}$ $=\int \frac{\frac{dx}{\cos^2 x}}{\frac{4\cos^2 x+9\sin^2 x}{\cos^2 x}}$ $=\int \frac{\sec^2 x dx}{4+9\tan^2 x}$ Put $\tan x = t$ $\sec^2 x\, dx = dt$ $=\int \frac{dt}{4+9t^2}$ $=\int \frac{dt}{(2)^2+(3t)^2}$ or $=\frac{1}{9}\int \frac{dt}{\left(\frac{2}{3}\right)^2+t^2}$ $=\frac{1}{2}\frac{\tan^{-1}\left(\frac{3t}{2}\right)}{3}+c$ or $=\frac{1}{9\left(\frac{2}{3}\right)}\tan^{-1}\left(\frac{t}{\frac{2}{3}}\right)+c$ $=\frac{1}{6}\tan^{-1}\left(\frac{3\tan x}{2}\right)+c$
105	$\int \frac{dx}{a^2\sin^2 x+b^2\cos^2 x}$ $=\int \frac{dx/\cos^2 x}{\frac{a^2\sin^2 x+b^2\cos^2 x}{\cos^2 x}}$ $=\int \frac{\sec^2 x dx}{a^2\tan^2 x+b^2}$ Put $\tan x = t$ $\therefore \sec^2 x dx = dt$ $=\int \frac{dt}{a^2 t^2+b^2}$ $=\frac{1}{b}\tan^{-1}\left(\frac{at}{b}\right)\frac{1}{a}+c$ $=\frac{1}{ab}\tan^{-1}\left(\frac{a\tan x}{b}\right)+c$

Thank You

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